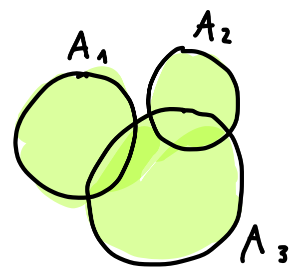
Konstrukcije na družinah

Družina množic $A : I \longrightarrow \mathrm{Set}$

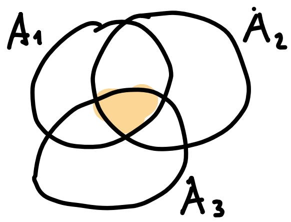
indeksna množica

Unija $\bigcup A$ $\qquad \bigcup_{i \in I} A_i$ $\qquad \sum_{n=1}^{100} n^2$

$$\bigcup_{i \in I} A_i = \{ x \mid \exists i \in I .\, x \in A_i \}$$



$$\bigcap_{i \in I} A_i = \{ x \mid \forall i \in I .\, x \in A_i \}$$



$X, Y \in \mathrm{Set}$ $\qquad X \cup Y = \bigcup_{i \in \{1,2\}} A_i$ $\qquad A_1 := X$, $A_2 := Y$

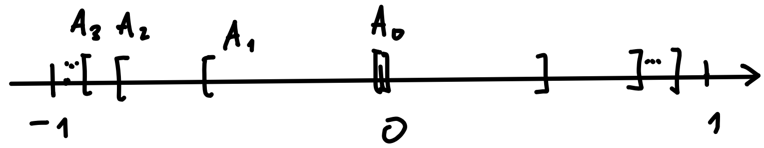
$\cap \qquad \bigcap$

$I = \mathbb{N}$ $\qquad A_i = [-1 + 2^{-i}, 1 - 2^{-i}]$

$A_0 = [0,0]$

$A_1 = [-\frac{1}{2}, \frac{1}{2}]$

$A_2 = [-\frac{3}{4}, \frac{3}{4}]$



$\bigcup A = (-1, 1)$

$\bigcap A = [0,0]$

$$\bigcup_{i\in I} A_i = \{x \mid \exists i \in I .\ x \in A_i\}$$

$\{x \mid \varphi(x)\}$ razred

<u>Aksiom o uniji</u>: Unija družine množic je množica.

Presek prazne družine: $A : \emptyset \to Set$

$$\bigcap_{i\in\emptyset} A_i = \{x \mid \forall i \in \emptyset .\ x \in A_i\}$$
$$= \{x \mid \top\}$$
$$= Set$$

pravi razred

Presek neprazne družine: $A : I \to Set$, $i_0 \in I$

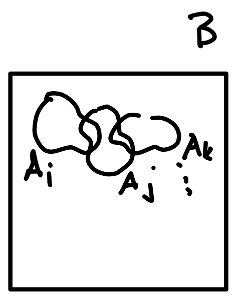
$$\bigcap_{i\in I} A_i = \{x \mid \forall i \in I .\ x \in A_i\} \subseteq A_{i_0}$$
$$= \{x \in A_{i_0} \mid \forall i \in I .\ x \in A_i\}$$

množica

Kadar imamo družino podmnožic množice B:

$$A : I \to \mathcal{P}(B)$$

vsak $A_i \subseteq B$



$$\bigcup A \subseteq B$$

Če je $I \neq \emptyset$: $\bigcap A \subseteq B$

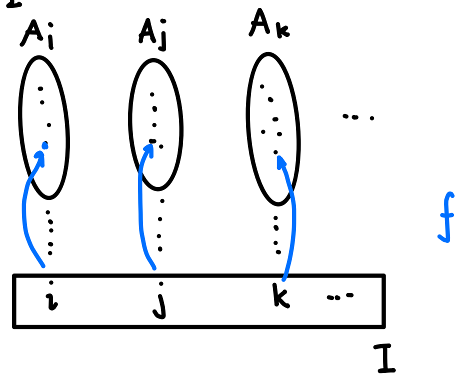
Če je $I = \emptyset$: $\bigcap A := B$

Presek družine <u>podmnožic</u>:

$$\bigcap_{i\in I} A_i := \{x \in B \mid \forall i \in I .\ x \in A_i\}$$

Kartezični produkt

$A : I \to \mathrm{Set}$



Def: Funkcija izbire f za družino $A : I \to \mathrm{Set}$ je tako prirejanje, ki vsakemu indeksu $i \in I$ priredi natanko en element $f(i) \in A_i$

Primer:

$I = \{1, 2, 3\}$

$A_1 := \{5, 6, 7\}$
$A_2 := \{6, 8\}$
$A_3 := \{1, 2, 10\}$

$f(1) := 6$
$f(2) := 6$
$f(3) := 1$

f je funkcija izbire za A.

$g(1) := 5$
$g(2) := 6$
$g(3) = 10$

Vseh je $3 \cdot 2 \cdot 3 = 18$.

Primer:

$I = \{1, 2, 3\}$

$A_1 := \{7\}$
$A_2 := \emptyset$
$A_3 := \{8, 9\}$

$f(1) := 7$
$f(2) :=$
$f(3) := 8$

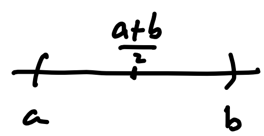
NI FUNKCIJE IZBIRE

Splošno: če $A : I \to \mathrm{Set}$ in $i_0 \in I$, $A_{i_0} = \emptyset$, potem A nima funkcije izbire.

Primer:

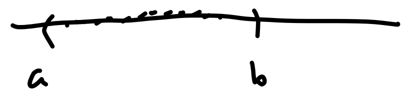
$I = \{(a,b) \in \mathbb{R} \times \mathbb{R} \mid a < b\}$

$A_{(a,b)} = \{x \in \mathbb{R} \mid a < x < b\}$



$f(a,b) := \frac{a+b}{2}$

$B_{(a,b)} := \{x \in \mathbb{Q} \mid a < x < b\}$



$g(a,b)$

$a = 3.712165999.6\ldots$

$b = 3.7121663$

$g(a,b) = 3.712165999..7000000$

<u>Definicija</u>: <u>Kartezični produkt</u> družine $A : I \to \mathrm{Set}$ je množica

$$\prod_{i \in I} A_i := \{ f : I \to \bigcup_{i \in I} A_i \mid \forall i \in I.\ f(i) \in A_i \}$$

"f je funkcija izbire za A"

Projekcije: za $j \in I$, $\quad pr_j : \prod_{i \in I} A_i \longrightarrow A_j$

$$pr_j(f) := f(j)$$

<u>Primer</u>

X, Y množici: $\quad I := \{1,2\} \quad A_1 := X, \quad A_2 := Y$

funkcija izbire: $\quad f : I \to X \cup Y$

$f(1) \in A_1 = X$

$f(2) \in A_2 = Y$

$$\prod_{i \in I} A_i \cong X \times Y$$

$$f \longmapsto (f(1), f(2))$$

$$\begin{pmatrix} 1 \mapsto u \\ 2 \mapsto v \end{pmatrix} \longleftarrow (u, v)$$

<u>Primer</u>: X, Y množici $\quad I := X \quad A_x := Y$ za $x \in X$

Konstantna družina

Funkcija izbire: $\quad f : X \to \bigcup A = Y \qquad f : X \to Y$

$\forall x \in X.\ f(x) \in A_x = Y \qquad \forall x \in X.\ f(x) \in Y$ itak velja

$$\prod_{x \in X} A_x = \{ f : X \to Y \mid T \} = Y^X$$

$$4 \cdot 4 \cdot 4 = 4^3$$

Koprodukt ali vsota množic

družina $A : I \to \mathrm{Set}$

$$\sum_{i \in I} A_i := \{ in_i(x) \mid i \in I,\ x \in A_i \}$$ je množica

Pišemo tudi $\coprod_{i \in I} A_i$

$$in_j : A_j \to \sum_{i \in I} A_i$$ injekcija

$$x \mapsto in_j(x)$$

projekciji

$$pr_1 : \sum_{i \in I} A_i \to I$$

$$in_i(x) \mapsto i$$

$$pr_2 : \sum_{i \in I} A_i \to \bigcup_{i \in I} A_i$$

$$in_i(x) \mapsto x$$

Primer: $I = \{1,2,3\}$

$A_1 := \{a, b, c\}$

$A_2 := \{a, d\}$

$A_3 := \{b\}$

$$\sum_{i \in I} A_i := \{ in_1(a), in_1(b), in_1(c), in_2(a), in_2(d), in_3(b) \}$$

Primer: $I = \{1,2\}$ $\quad A_1 := X$

$A_2 := Y$

$$\sum_{i \in I} A_i = X + Y$$

<u>Primer</u>: X, Y množici $\qquad I = X \qquad A_x := Y$

$$\sum_{x \in X} A_i := \{ in_x(y) \mid x \in X, y \in Y \} \cong X \times Y$$

$$in_x(y) \longmapsto (x, y)$$

$$in_u(v) \longleftarrow\!\!\!\!\mid (u, v)$$

$3 + 3 + 3 + 3 \qquad\qquad 4 \times 3$