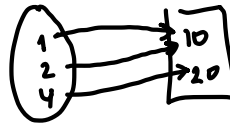# Preslikava / funkcija

$$f : A \longrightarrow B$$

prirejanje   domena   kodomena

↳ celovito: vsakemu $x \in A$ priredi vsaj en element B

↳ enolično: če priredi $x \in A$ hkrati $y \in B$ in $z \in B$, potem $y = z$.

Primer: $\{1, 2, 4\} \rightarrow \{10, 20\}$

$$
\begin{aligned}
1 &\longmapsto 10 \\
2 &\longmapsto 10 \\
4 &\longmapsto 20
\end{aligned}
$$

1	10
2	10
4	20

Funkcijski predpis:

$$x \longmapsto \text{izraz, ki lahko vsebuje } x$$

$$\mathbb{R} \longrightarrow \mathbb{R}$$
$$x \longmapsto \sin(x^2) + \sqrt[3]{x}$$
$$x \longmapsto \sin(x^2) + \sqrt[2]{x} \quad \longleftarrow \quad \sqrt{} \text{ ni definiran za } x < 0 \text{ (je v } \mathbb{C}, \text{ ampak kodomena je } \mathbb{R})$$

$$A \times B \longrightarrow A \times A$$
$$u \longmapsto (pr_1(u), pr_1(u)) \quad \checkmark$$
$$(x, y) \longmapsto (x, x) \quad \checkmark$$

$$\mathbb{R} \longmapsto \mathbb{R}$$
$$x + y \longmapsto x - y \quad ?? \quad \textcolor{red}{\times}$$

$$7 = 3 + 4 \qquad 3 + 4 \longmapsto 3 - 4 \qquad 7 \longmapsto -1$$
$$7 = 2 + 5 \qquad 2 + 5 \longmapsto 2 - 5 \qquad 7 \longmapsto -3 \qquad \text{NI ENOLIČNO}$$

$$A \longrightarrow B$$
$$x \longmapsto y \quad ?$$

Preslikave $\emptyset \to A$

Logika:

"Za vse $x \in \emptyset$ velja ....."   VEDNO RES

"Obstaja $x \in \emptyset$, za katerega velja ....."   VEDNO NERESNIICA

Celovitost: $\underline{za \text{ } vsak \text{ } x \in \emptyset}$, prirejanje x-u priredi vsaj
                $\checkmark$ en element iz A

Enoličnost: $\underline{za \text{ } vsak \text{ } x \in \emptyset}$, če x-u priredimo y in z, potem $y = z$.
                   $\checkmark$

$f : \emptyset \to A$ $g : \emptyset \to A$

$f = g$ če   za vsak $\underline{x \in \emptyset}$ velja $f(x) = g(x)$.
                 $\checkmark$

    Prazna preslikava: $\emptyset \to A$

Preslikava $A \to \emptyset$?

1) Če $A = \emptyset$:   prazna preslikava $\emptyset \to \emptyset$

2) Če $A \neq \emptyset$, je prav, obstaja $a \in A$
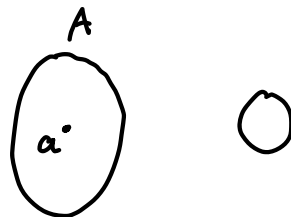
                     A NI PRESLIKAVE

Identiteta

    $id_A : A \to A$

    $id_A : x \mapsto x$ $id_A(x) := x$ $id_A(x) = x$

Kompozitum:

$$A \xrightarrow{f} B \xrightarrow{g} C$$
$$g \circ f$$

$$g \circ f : A \longrightarrow C$$
$$ x \longmapsto g(f(x))$$
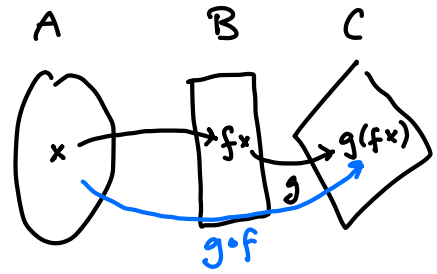
$$(g \circ f)(x) := g(f(x))$$

$$A \xrightarrow{id_A} A \xrightarrow{f} B$$
$$f \circ id_A$$

$$f \circ id_A : A \rightarrow B$$
$$f \circ id_A = f$$

Naj bo $x \in A$: $\quad (f \circ id_A)(x) = f(id_A(x))$
$$= f(x)$$

$$id_B \circ f = f$$

Funkcijski predpisi na $A \times B$ in $A + B$:

$$\mathbb{R} \times \mathbb{Z} \longrightarrow \mathbb{R}$$
$$p \longmapsto pr_1(p) + pr_2(p)^2$$
$$(3,7) \longmapsto pr_1(3,7) + pr_2(3,7)^2 = 3 + 7^2 = 52$$
$$(x,y) \longmapsto x + y^2$$

$$A \times B \times C \longmapsto D$$
$$(x,y,z) \longmapsto \cdots \checkmark$$
$$A B C$$

$$(A \times B) \times C \longrightarrow D$$
$$((x,y),z) \longmapsto \cdots \checkmark$$
$$A B C$$
$$(p_{A \times B}, z_C) \longmapsto \cdots \checkmark$$

$A + B \longrightarrow C$?

$in_1(x) \longmapsto \cdots$

$in_2(y) \longmapsto \cdots$

$\mathbb{R} + \mathbb{Z} \longrightarrow \mathbb{R}$

budisi $\quad in_1(x) \longmapsto x^2 + 7$
$\qquad \underset{\mathbb{R}}{}$

budisi $\quad in_2(y) \longmapsto 3y - 6$
$\qquad \underset{\mathbb{Z}}{}$

$in_1(8) \longmapsto 8^2 + 7$

$in_2(5) \longmapsto 3 \cdot 5 - 6$

$in_1(x) \longmapsto x^2 + 7 \quad \checkmark$

$in_2(x) \longmapsto x^2 + 7$

$in_1(x) \longmapsto x^2 + 7$

$in_2(x) \longmapsto 3x - 6$

$\mathbb{R} \times \mathbb{Z} \longrightarrow \mathbb{R}$

$(x, x) \longmapsto x^2 + 7$? ✗

$(7, 7) \longmapsto 7^2 + 7$

$(\pi/6, 8) \longmapsto$?

$f : \mathbb{R} + \mathbb{Z} \longrightarrow \mathbb{R}$

$f : in_1(x) \longmapsto x^2$

$f : in_2(y) \longmapsto 3y$

ali

$f(in_1(x)) := x^2$

$f(in_2(y)) := 3y$

$(\mathbb{R} \times \mathbb{R}) + \mathbb{Z} \longrightarrow \mathbb{R}$

$in_1(x, y) \longmapsto 2x^2 + y$

$in_2(k) \longmapsto \sin\left(\frac{\pi k}{4}\right)$

$(\mathbb{R} + \mathbb{R}) \times \mathbb{Z} \longrightarrow \mathbb{R}$

$(\underbrace{in_1(x)}_{\mathbb{R}+\mathbb{R}}, \underbrace{k}_{\mathbb{Z}}) \longmapsto x^2 + k$

$(in_2(y), m) \longmapsto y + m$

Eksponentne množice in preslikave:

B^A množica vseh preslikav iz A v B

$\gamma \equiv$

$\psi : \mathbb{R}^{\mathbb{R}} \longrightarrow \mathbb{R}$

$\psi : f \longmapsto f(0) + f(1)$ $\qquad \psi(f) := f(0) + f(1)$

$\psi(x \mapsto x^2 + 3) = (x \mapsto x^2 + 3)(0) + (x \mapsto x^2 + 3)(1)$

$\qquad\qquad = (0^2 + 3) + (1^2 + 3) = 7$

OPOZORILO:

$\begin{array}{l} 3 \\ \sqrt{5} \\ x \in \mathbb{R}, \quad \sqrt{x} \longrightarrow \text{izraz, število} \\ \quad x \mapsto \sqrt{x} \quad \text{predpis} \end{array}$

Evalvacija / aplikacija / uporaba:

$ev : B^A \times A \longrightarrow B$

$ev(\underset{B^A}{f}, \underset{A}{x}) := \underset{B}{f(x)}$

$A \xrightarrow{f \in B^A} B \xrightarrow{g \in C^B} C$

$\underset{g \circ f}{}$

Kompozitum:

$\circ : C^B \times B^A \longrightarrow C^A$

$\circ : (g, f) \longmapsto (x \mapsto g(f(x)))$

Primer:

$\Phi : \mathbb{R} \longrightarrow \mathbb{R}^{\mathbb{R}}$
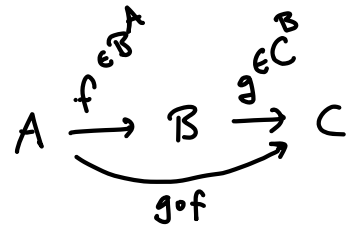
$\qquad x \longmapsto (y \mapsto 2x + 3y)$
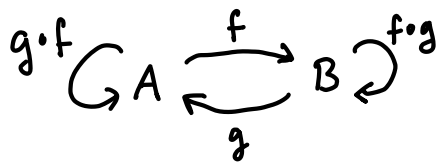$\qquad \underset{\mathbb{R}}{} \qquad \underset{\mathbb{R}^{\mathbb{R}}}{}$

$? \longrightarrow x \mapsto (x \mapsto 2x + 3x)$ ZMEDA

$\Phi(7) = (y \mapsto 2 \cdot 7 + 3y)$

"$\Phi(7)$ je funkcija, ki število množi s 3 in prišteje 14"

$\left(\Phi(7)\right)(5) = (y \mapsto 2 \cdot 7 + 3y)(5) = 2 \cdot 7 + 3 \cdot 5$

Preslikava $g: B \to A$ je _inverz_ preslikave $f: A \to B$,
ko velja: $g \circ f = id_A$ in $f \circ g = id_B$.

$$g \circ f \quad \curvearrowright A \underset{g}{\overset{f}{\rightleftarrows}} B \quad \circlearrowright^{f \circ g}$$

Primer :

$$f: \mathbb{R} \longrightarrow \mathbb{R}_{\geq 0} \qquad g: \mathbb{R}_{\geq 0} \to \mathbb{R}$$
$$x \longmapsto x^2 \qquad\qquad x \longmapsto \sqrt{x}$$

za $x \geq 0$: $f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x$ NISTA

za $x \in \mathbb{R}$ $g(f(x)) = g(x^2) = \sqrt{x^2} = |x|$ INVERZA

Primer :

$$f: \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0} \quad f(x) = x^2$$
$$g: \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0} \quad g(x) = \sqrt{x}$$

STA INVERZA.

Če ima $f: A \to B$ inverza $\underbrace{g, h: B \to A}_{\text{okrajšava za}}$, potem sta

g in h enaka : $g: B \to A$ in $h: B \to A$

Vemo : $f \circ g = id_B$ ⌈ Naj bosta $x, y \in \mathbb{R}$ .....

$g \circ f = id_A$ (2)   ⌊ Naj bo $x \in \mathbb{R}$ in $y \in \mathbb{R}$ ....

$f \circ h = id_B$ (3)

$h \circ f = id_A$

$$g = g \circ id_B \overset{(3)}{=} g \circ (f \circ h) \overset{\text{asociativnost}}{=} (g \circ f) \circ h = id_A \circ h = h.$$

Kadar f ima inverz, ga označimo z f^{-1}.

Kadar f ima inverz, pravimo, da je _izomorfizem_.

Če je $f: A \to B$ izomorfizem, pravimo, da sta A in B _izomorfni_ množici in pišemo: $A \cong B$.

Primer:

$A \cong A$ ker $id_A : A \to A$ je izomorfizem

$$id_A^{-1} = id_A$$

$A \times B \cong B \times A$ ker $f : A \times B \to B \times A$ $g : B \times A \to A \times B$

$$(x,y) \mapsto (y,x) \qquad\qquad (u,v) \mapsto (v,u)$$

$$A \times (B+C) \cong A \times B + A \times C$$

$$A^{B+C} \cong A^B \times A^C \qquad \dots$$