

Ponovimo:

• preslikava $f: A \rightarrow B$
 $x \mapsto \dots$

• kartezični produkt $A \times B$

(a, b) urejeni par

$(a, b) \in A \times B$ ko je $a \in A$ in $b \in B$

Kanonične projekcije:

$$\pi_1: A \times B \rightarrow A$$

$$(x, y) \mapsto x$$

$$\pi_2: A \times B \rightarrow B$$

$$(x, y) \mapsto y$$

Ostale oznake za π_1 so: p_1, pr_1, fst, π_0

π_2 so: p_2, pr_2, snd, π_1

Kartezični produkt $A \times B \times C$:

urejene trojice $(a, b, c) \in A \times B \times C$ kjer $A \ni a$

$b \in B$

$c \in C$

projekcije: $\pi_1: A \times B \times C \rightarrow A$

$$\pi_2: A \times B \times C \rightarrow B, (x, y, z) \mapsto y$$

$$\pi_3: A \times B \times C \rightarrow C$$

Kartezični produkt ene množice A ? To je kar A

(Python: $(a,)$ urejena enaterica)

Kartezični produkt nič množic?
 (a, b, c, d) četverica
 $()$ ničtenica
 Standardni enojec!
 $1 = \{()\}$

Vsota množic

Tudi: disjunktna unija, koproduct, disjunktna vsota

$A + B$ elementi so

$l_1(a)$ kjer $a \in A$

$l_2(b)$ kjer $b \in B$

Primer:

$$\{1, 2\} + \{2, 3, 4\} = \{l_1(1), l_1(2), l_2(2), l_2(3), l_2(4)\}$$

Kanonični injekciji

$l_1: A \rightarrow A+B$
 $a \mapsto l_1(a)$
 ↗ preslikava
 ↖ oznaka

$l_2: B \rightarrow A+B$

$b \mapsto l_2(b)$

$a \mapsto l_2(a)$

← tudi
 OK,
 neujedno

$A+B+C$ elementi

$l_1(a)$ $a \in A$

$l_2(b)$ $b \in B$

$l_3(c)$ $c \in C$

Vsota nič množic je \emptyset

Kako definiramo preslikavo

$$\begin{array}{l} A + B \longrightarrow C \quad ? \\ L_1(a) \longmapsto \dots a \dots \\ L_2(b) \longmapsto \dots b \dots \end{array} \left. \vphantom{\begin{array}{l} A + B \longrightarrow C \\ L_1(a) \longmapsto \dots a \dots \\ L_2(b) \longmapsto \dots b \dots \end{array}} \right\} \text{predpis ima} \\ \text{dva primera}$$

Tudi:

$$x \longmapsto \begin{cases} \dots a \dots & \text{če } x = L_1(a) \\ \dots b \dots & \text{če } x = L_2(b) \end{cases}$$

Primeri:

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

$$f(x) = \begin{cases} -x & \text{če } x \leq 0 \\ x & \text{če } x \geq 0 \end{cases}$$

Kompozitum & identiteta

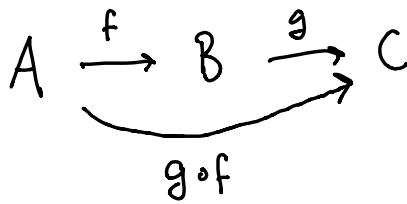
Identiteta na množici A :

$$\text{id}_A: A \rightarrow A$$

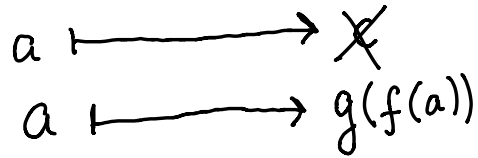
$$x \longmapsto x$$

$$\text{id}_A(x) = x$$

Kompozitum:



(računalnični
fig)



$$\mathbb{N} \xrightarrow{f} \mathbb{N} \xrightarrow{g} \mathbb{Z}$$

$$f(n) = 2n + 1$$

$$g(k) = k - 4$$

$$\mathbb{N} \xrightarrow{g \circ f} \mathbb{Z}$$

$g \circ f$: $5 \mapsto 7$

$x \mapsto 2x - 3$

Vprašanje:

$$A \times A \times A \longrightarrow A$$

koliko je kanoničnih
projekcij

$$\pi_1: (a, \hat{a}, \tilde{a}) \mapsto a$$

$$\pi_2: (a', y, a'') \mapsto y$$

$$\pi_3: (a', a'', \check{c}) \mapsto \check{c}$$

nevljudne
oznake

$$\pi_2(\square, \heartsuit, \diamond) = \heartsuit$$

$$\pi_3(\square, \heartsuit, \diamond) = \diamond$$

$$\square \in A, \heartsuit \in A, \diamond \in A$$

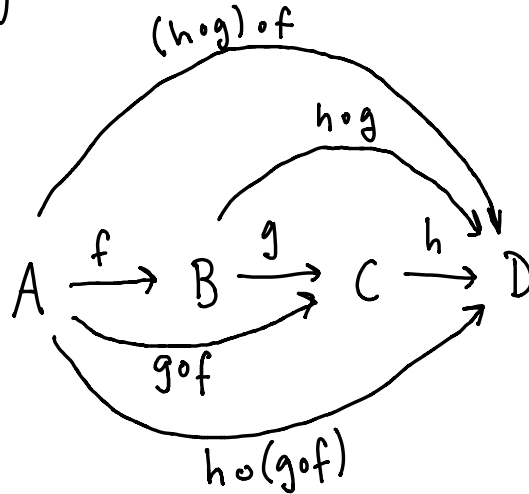
π_2 in π_3 nista enaki projekciji,
razen v primeru $A = \emptyset$ in
ko je A enojec

$$\{\star\} \times \{\star\} \times \{\star\} = \{(\star, \star, \star)\}$$

a a a

$$(a+b)+c = a+(b+c)$$

Kompozitum je asociativno:



$$(u \circ v)(r) = u(v(r))$$

$$(h \circ g) \circ f = h \circ (g \circ f)$$

Naj bo $x \in A$ poljubno. Pravinimo:

$$\underbrace{((h \circ g) \circ f)}_u(x) = \underbrace{(h \circ g)}_u(\underbrace{f(x)}_r) = \underbrace{h(g(f(x)))}_{\text{ok!}}$$

$$\underbrace{(h \circ (g \circ f))}_u(x) = \underbrace{h}_{\tilde{u}}(\underbrace{(g \circ f)}_r(x)) = \underbrace{h(g(f(x)))}_{\tilde{u}}$$

Identiteta je neutralni element za kompozitum:

$$A \xrightarrow{f} B$$

$$f \circ \text{id}_A = f$$

$$\text{id}_B \circ f = f$$

$$(f \circ \text{id}_A)(x) = f(\text{id}_A(x)) = f(x)$$

Izomorfizmi množic

$$\mathbb{N} \times \mathbb{Z} \neq \mathbb{Z} \times \mathbb{N}$$

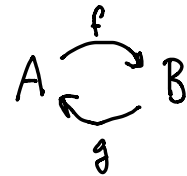
$$(1, -1) \in \mathbb{N} \times \mathbb{Z}$$

$$(1, -1) \notin \mathbb{Z} \times \mathbb{N}$$

Definicija: Naj bosta A in B množici ter $f: A \rightarrow B$ in $g: B \rightarrow A$.
Pravimo da je g inverz f , če velja

$$f \circ g = \text{id}_B$$

$$g \circ f = \text{id}_A$$



Definicija: Preslikava $f: A \rightarrow B$ je izomorfizem,
če obstaja njen inverz.

Definicija: Množici X in Y sta izomorfni, če
obstaja izomorfizem $X \rightarrow Y$. Pišemo $X \cong Y$.

Primer:

$$\mathbb{N} \times \mathbb{Z} \cong \mathbb{Z} \times \mathbb{N} ?$$

izomorfizem $f: \mathbb{N} \times \mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{N} ?$

$$f: \mathbb{N} \times \mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{N} \text{ in } g: \mathbb{Z} \times \mathbb{N} \rightarrow \mathbb{N} \times \mathbb{Z} ?$$

$$f \circ g = \text{id}_{\mathbb{Z} \times \mathbb{N}} \text{ in } g \circ f = \text{id}_{\mathbb{N} \times \mathbb{Z}}$$

Tak f obstaja!


$$f: \mathbb{N} \times \mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{N}$$

$$f: (x, y) \mapsto (y, x)$$

$$g: \mathbb{Z} \times \mathbb{N} \rightarrow \mathbb{N} \times \mathbb{Z}$$

$$g: (\vartheta, \psi) \mapsto (\psi, \vartheta)$$

xi
zeta



Preverimo: $f \circ g = \text{id}_{\mathbb{Z} \times \mathbb{N}}$ Naj bo $a \in \mathbb{N}$ in $b \in \mathbb{Z}$.

$$(f \circ g)(b, a) = f(g(b, a)) = f(a, b) = (b, a) \quad \checkmark$$

Naj bo $\alpha \in \mathbb{N}$ in $\beta \in \mathbb{Z}$. Preverimo:

$$(g \circ f)(\alpha, \beta) = g(f(\alpha, \beta)) = g(\beta, \alpha) = (\alpha, \beta) \quad \checkmark$$

...

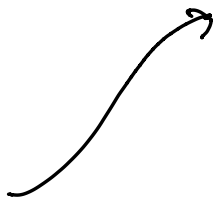
$$A \times B \cong B \times A$$

$$A \cong A$$

$$A \times 1 \cong A$$

}

1. $A + \emptyset \cong A$
2. $A + B \cong B + A$
3. $(A + B) + C \cong A + (B + C)$
4. $A \times 1 \cong A$
5. $A \times B \cong B \times A$
6. $(A \times B) \times C \cong A \times (B \times C)$
7. $A \times (B + C) \cong A \times B + A \times C$
8. $A \times \emptyset \cong \emptyset$
9. $A^1 \cong A$
10. $1^A \cong 1$
11. $A^\emptyset \cong 1$
12. $\emptyset^A \cong \emptyset$ če $A \neq \emptyset$
13. $A^{(B \times C)} \cong (A^B)^A$
14. $A^{(B + C)} \cong A^B \times A^C$
15. $(A \times B)^C \cong A^C \times B^C$



$$f: A^{B \times C} \rightarrow (A^B)^C$$

$$f: h \mapsto (x \mapsto (y \mapsto h(x, y)))$$

$$f^{-1}: (A^B)^C \rightarrow A^{B \times C}$$

$$f^{-1}: g \mapsto ((u, v) \mapsto g(u)(v))$$

$$f \circ f^{-1} = \text{id}_{(A^B)^C}$$

$$f^{-1} \circ f = \text{id}_{A^{B \times C}}$$