Ponovimo:

- preslikava $f : A \longrightarrow B$

  $x \longmapsto \ldots$

- kartezični produkt $A \times B$

  (a,b) urejeni par

  $(a,b) \in A \times B$ ko je $a \in A$ in $b \in B$

Kanonične projekcije:

$$\pi_1 : A \times B \longrightarrow A \qquad \pi_2 : A \times B \longrightarrow B$$
$$(x,y) \longmapsto x \qquad (x,y) \longmapsto y$$

Ostale oznake za π_1 so: p_1, pr_1, fst, π_0

π_2 so: p_2, pr_2, snd, π_1

Kartezični produkt $A \times B \times C$:

urejene trojice $(a,b,c) \in A \times B \times C$ kjer $A \ni a$

$b \in B$

$c \in C$

projekcije: $\pi_1 : A \times B \times C \longrightarrow A$

$\pi_2 : A \times B \times C \longrightarrow B$, $(x,y,z) \longmapsto y$

$\pi_3 : A \times B \times C \longrightarrow C$

Kartezični produkt ene množice A? To je kar A

(Python: $(a,)$ urejena enaterica)

Kartezični produkt nič množic? (a,b,c,d) četverica

$()$ ničterica

Standardni enojec! $1 = \{()\}$

Vsota množic

Tudi: disjunktna unija, koprodukt, disjunktna vsota

$A + B$ elementi so

$\iota_1(a)$ kjer $a \in A$

$\iota_2(b)$ kjer $b \in B$

Primer:

$$\{1,2\} + \{2,3,4\} = \{\iota_1(1), \iota_1(2), \iota_2(2), \iota_2(3), \iota_2(4)\}$$

Kanonični injekciji

$\iota_1 : A \longrightarrow A+B$ (preslikava)

$a \longmapsto \iota_1(a)$ (oznaka)

$\iota_2 : B \longrightarrow A+B$

$b \longmapsto \iota_2(b)$

$a \longmapsto \iota_2(a)$ ← tudi OK, nerodno

$A+B+C$ elementi $\iota_1(a)$ $a \in A$

$\iota_2(b)$ $b \in B$

$\iota_3(c)$ $c \in C$

Vsota nič množic je $\emptyset$

Kako definiramo preslikavo

$$A + B \longrightarrow C \quad ?$$

$$\left.\begin{array}{l} \iota_1(a) \longmapsto \cdots\cdots a \cdots\cdots \\ \iota_2(b) \longmapsto \cdots\cdots b \cdots\cdots \end{array}\right\}$$ predpis ima <u>dva</u> primera

Tudi:

$$x \longmapsto \begin{cases} \cdots a \cdots & \text{če } x = \iota_1(a) \\ \cdots b \cdots & \text{če } x = \iota_2(b) \end{cases}$$

Primer:

$$f : \mathbb{R} \to \mathbb{R}$$

$$f(x) = \begin{cases} -x & \text{če } x \le 0 \\ x & \text{če } x \ge 0 \end{cases}$$

Kompozitum & identiteta

Identiteta na množici A:

$$\mathrm{id}_A : A \longrightarrow A$$
$$x \longmapsto x$$

$$\mathrm{id}_A(x) = x$$

Kompozitum:

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[rr, bend right, "g\circ f"'] & B \arrow[r, "g"] & C \end{tikzcd}$$

(računalničarji $f;g$)

$a \longmapsto \cancel{c}$

$a \longmapsto g(f(a))$

$$\mathbb{N} \xrightarrow{f} \mathbb{N} \xrightarrow{g} \mathbb{Z}$$

$f(n) = 2n+1$

$g(k) = k-4$

$$\mathbb{N} \xrightarrow{g\circ f} \mathbb{Z}$$

$g\circ f$: $5 \longmapsto 7$

$x \longmapsto 2x-3$

Vprašanje:

$A \times A \times A \longrightarrow A$ Koliko je kanoničnih projekcij

$\pi_1 : (a, \hat{a}, \tilde{a}) \longmapsto a$

$\pi_2 : (a', y, a''') \longmapsto y$

$\pi_3 : (a', a''', \check{c}) \longmapsto \check{c}$

nevljudne oznake

$\pi_2(\square, \heartsuit, \diamondsuit) = \heartsuit$

$\pi_3(\square, \heartsuit, \diamondsuit) = \diamondsuit$

$\square \in A$, $\heartsuit \in A$, $\diamondsuit \in A$

π_2 in π_3 nista enaki preslikavi, razen v primeru $A = \emptyset$ in ko je A enojec

$\{\star\}\times\{\star\}\times\{\star\} = \{(\star,\star,\star)\}$ $\quad a \quad a \quad a$

$(a+b)+c = a+(b+c)$

Kompozitum je asociativen:

$$\begin{tikzcd}
A \arrow[r, "f"] \arrow[rr, bend right, "g\circ f"'] \arrow[rrr, bend left=60, "(h\circ g)\circ f"] \arrow[rrr, bend right=60, "h\circ(g\circ f)"'] & B \arrow[r, "g"] \arrow[rr, bend left, "h\circ g"] & C \arrow[r, "h"] & D
\end{tikzcd}$$

$(u\circ v)(r) = u(v(r))$

$$(h\circ g)\circ f = h\circ(g\circ f)$$

Naj bo $x\in A$ poljuben. Preverimo:

$$(\underbrace{(h\circ g)}_{u}\circ\underbrace{f}_{v})(\underbrace{x}_{r}) = \underbrace{(h\circ g)}_{u}(\underbrace{f}_{v}(\underbrace{x}_{r})) = h(g(f(x))$$

$\parallel$ ok!

$$(\underbrace{h}_{u}\circ\underbrace{(g\circ f)}_{v})(\underbrace{x}_{r}) = \underbrace{h}_{u}(\underbrace{(g\circ f)}_{v}(\underbrace{x}_{r})) = h(g(f(x)))$$

Identiteta je nevtralni element za kompozitum:

$$A \xrightarrow{f} B$$

$$f\circ id_A = f$$

$$id_B\circ f = f$$

$(f\circ id_A)(x) = f(id_A(x))$
$= f(x)$

Izomorfizmi množic

$$\mathbb{N} \times \mathbb{Z} \neq \mathbb{Z} \times \mathbb{N}$$

$(1,-1) \in \mathbb{N} \times \mathbb{Z}$ $\qquad$ $(1,-1) \notin \mathbb{Z} \times \mathbb{N}$

Definicija: Naj bosta A in B množici ter $f: A \to B$ in $g: B \to A$. Pravimo da je g <u>inverz</u> f, če velja

$$f \circ g = id_B \quad \text{in}$$
$$g \circ f = id_A.$$

$$\begin{tikzcd} A \arrow[r, bend left, "f"] & B \arrow[l, bend left, "g"] \end{tikzcd}$$

Definicija: Preslikava $f: A \to B$ je <u>izomorfizem</u>, če obstaja njen inverz.

Definicija: Množici X in Y sta <u>izomorfni</u>, če obstaja izomorfizem $X \to Y$. Pišemo $X \cong Y$.

<u>Primer:</u> $\mathbb{N} \times \mathbb{Z} \cong \mathbb{Z} \times \mathbb{N}$?

izomorfizem $f: \mathbb{N} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{N}$?

$f: \mathbb{N} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{N}$ in $g: \mathbb{Z} \times \mathbb{N} \to \mathbb{N} \times \mathbb{Z}$?

$f \circ g = id_{\mathbb{Z} \times \mathbb{N}}$ in $g \circ f = id_{\mathbb{N} \times \mathbb{Z}}$

Tak f obstaja!

xi
zeta

$$f : \mathbb{N} \times \mathbb{Z} \longrightarrow \mathbb{Z} \times \mathbb{N}$$

$$f : (x, y) \longmapsto (y, x)$$

$$g : \mathbb{Z} \times \mathbb{N} \longrightarrow \mathbb{N} \times \mathbb{Z}$$

$$g : (\vartheta, \Upsilon) \longmapsto (\Upsilon, \vartheta)$$

Preverimo: $f \circ g = id_{\mathbb{Z} \times \mathbb{N}}$ Naj bo $a \in \mathbb{N}$ in $b \in \mathbb{Z}$.

$$(f \circ g)(b, a) = f(g(b, a)) = f(a, b) = (b, a) \quad \checkmark$$

Naj bo $\alpha \in \mathbb{N}$ in $\beta \in \mathbb{Z}$. Preverimo:

$$(g \circ f)(\alpha, \beta) = g(f(\alpha, \beta)) = g(\beta, \alpha) = (\alpha, \beta) \quad \checkmark$$

— · —

$$A \times B \cong B \times A$$

$$A \cong A$$

$$A \times 1 \cong A$$

1. $A + \varnothing \cong A$
2. $A + B \cong B + A$
3. $(A + B) + C \cong A + (B + C)$

4. $A \times 1 \cong A$
5. $A \times B \cong B \times A$
6. $(A \times B) \times C \cong A \times (B \times C)$

7. $A \times (B + C) \cong A \times B + A \times C$
8. $A \times \varnothing \cong \varnothing$

9. $A^1 \cong A$
10. $1^A \cong 1$
11. $A^\varnothing \cong 1$
12. $\varnothing^A \cong \varnothing$ če $A \neq \varnothing$
13. $A^{(B \times C)} \cong (A^B)^C$
14. $A^{(B + C)} \cong A^B \times A^C$
15. $(A \times B)^C \cong A^C \times B^C$

$$f : A^{B \times C} \longrightarrow (A^B)^C$$

$$f : h \longmapsto (x \mapsto (y \mapsto h(x,y)))$$

$$f^{-1} : (A^B)^C \longrightarrow A^{B \times C}$$

$$f^{-1} : g \longmapsto ((u,v) \mapsto g(u)(v))$$

$$f \circ f^{-1} = id_{(A^B)^C}$$

$$f^{-1} \circ f = id_{A^{B \times C}}$$