λ - račun

α β γ δ η eta ϑ theta ι iota χ HI φ π ξ KSI ζ ZETA

Java:

- ~~razredi~~
- ~~objekti~~
- ~~tabele~~
- ~~String~~
- spremenljivke
- ~~int~~
- ~~float~~

- ~~tipi~~
- ~~if-then-else~~
- ~~+ - x~~
- ~~while~~
- ~~for~~
- ~~izjeme~~
- funkcije - ~~metode~~
- ~~rekurzija~~

if b then e_1 else e_2

$b \cdot e_1 + (1-b)\, e_2$

Funkcijski predpis:

$$x \mapsto x^2 + 3x - 7$$

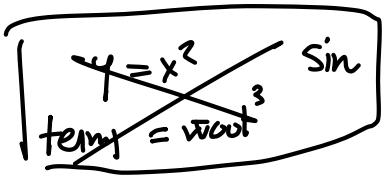
"x se slika v x^2+3x-7"

$f(x) := x^2 + 3x - 7$ POIMENOVANA FUNKCIJA

$x \mapsto x^2 + 3x - 7$ NEIMENOVANA / ANONIMNA FUNKCIJA

42 ŠTEVILO

$a := 42$ IMENOVANO ŠTEVILO



$f := (x \mapsto x^2 + 3x - 7)$

$x \mapsto x^2 + 3x - 7$

$f(x) = x^2 + 3x - 7$

$f(7) \rightsquigarrow 7^2 + 3 \cdot 7 - 7$

$\downarrow$ račun

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$(x \mapsto x^2 + 3 \cdot x - 7)(7) \rightsquigarrow 7^2 + 3 \cdot 7 - 7$

$(A \mapsto A^2 + 3 \cdot A - 7)(7)$

$(f \mapsto f(18))\ (a \mapsto a+3) \rightsquigarrow \underbrace{(a \mapsto a+3)(18)}_{f} \rightsquigarrow 18+3$

vgnezden predpis

$(x \mapsto (y \mapsto 3 \cdot x^2 \cdot y + y - x))(6) \rightsquigarrow$

$y \mapsto 3 \cdot 6^2 \cdot y + y - 6$

$((x \mapsto (y \mapsto 10 \cdot x + y))(6))(2) \rightsquigarrow$

$(y \mapsto 10 \cdot 6 + y)(2) \rightsquigarrow$

$10 \cdot 6 + 2$

$\langle \text{izraz} \rangle(6)(2)$

$[(\text{izraz})(6)](2)$

Proste & vezane spremenljivke

$x \mapsto a^2 \cdot x$ "množi z a^2"

$a \mapsto a^2 \cdot x$ "kvadriraj in pomnoži z x"

$x \mapsto a^2 \cdot x$

x je VEZANA v tem predpisu

a: prosta

$$\int_0^1 \frac{1}{a + x^2}\, dx$$

x: VEZANA

a: prosta (prosti parameter)

$$\forall x \in \text{Human}.\ \exists y \in \text{Human}.\ \text{Loves\_sometimes}(x, y)$$

x, y: vezani

$$\exists y \in \text{Human}.\ \text{Loves\_sometimes}(x, y)$$

x: prosta

$$\int_0^1 \frac{1}{a + x^2}\, dx = \int_0^1 \frac{1}{a + t^2}\, dt \neq \int_0^1 \frac{1}{a + a^2}\, da$$

?!

"a smo ujeli"

Zamenjava ali substitucija

"V izrazu e zamenjaj spremenljivko x z e'"

$e[e'/x]$

pišemo tudi

$e[x \to e']$

$(a^2+b^2)[7/a] = 7^2+b^2$

$(a^2+b^2)[b/a] = b^2+b^2$

$(a^2+b^2)[7/c] = a^2+b^2$

$(f(f(8)))[(x \mapsto x+6)/f] =$

$((x \mapsto x+6)((x \mapsto x+6)(8)))$

$(x \mapsto a^2+3\cdot x)[(16+y)/a] = (x \mapsto (16+y)^2+3x)$

$(x \mapsto a^2+3\cdot x)[(16+x)/a] =$

PRAVILNO: $= (z \mapsto a^2+3\cdot z)[(16+x)/a] =$

$(z \mapsto (16+x)^2+3\cdot z)$

NAROBE: $= (x \mapsto (16+x)^2+3\cdot x)$

RAČUNSKO PRAVILO β-REDUKCIJE:

$(x \mapsto \underbrace{x^3+6}_{\text{telo funkcije}})(a+7) = (a+7)^3+6$

"V telesu funkcije x zamenjaj z $a+7$."

β-pravilo

$$(x \mapsto e)(e') = e[e'/x]$$

Churchev zapis funkcijskega pravila:

$$x \mapsto e$$

Church: $\lambda x . e$

Primer:

$x \mapsto 3x^2$	$\lambda x . 3x^2$
$x \mapsto (y \mapsto 6xy^2)$	$\lambda x . \lambda y . 6xy^2$
	$\lambda xy . 6xy^2$ okrajšava

$\hat{x}$ — $\hat{x}.e$ — $\hat{}x.e$ — $\lambda x.e$

$\iota x . \varphi$	"tisti x, ki zadošča φ"	Russell
$\varepsilon x . \varphi$	"kateriKoli x, ki zadošča φ"	Hilbert
$\lambda x . e$		Church

$f(x)$
$f\ x$
$\sin \alpha$

λ-račun

Sintaksa izrazov:

- spremenljivke — $x, y, z,$ cow, channel, ...
- abstrakcija — $\lambda x . e$ — "x se slika v e"
- uporaba ali aplikacija — $e_1\ e_2$ — "e_1 uporabi na e_2" / "e_1 od e_2"

Računsko pravilo: β-redukcija

$$(\lambda x.e)\ e' = e[e'/x]$$

Uporaba veže levo:

$$e_1\ e_2\ e_3 = (e_1\ e_2)\ e_3$$

λ-abstrakcija veže, kolikor je mogoče

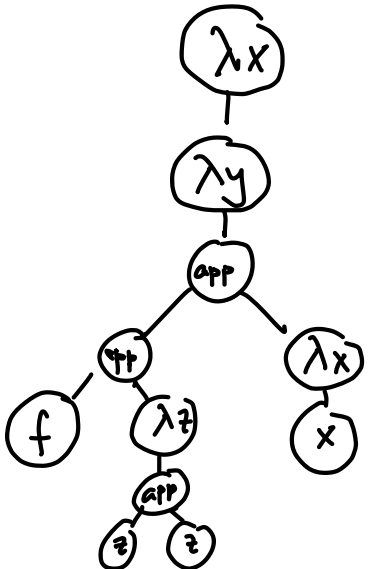
$$\lambda x.e_1\ e_2\ e_3$$

$$(\lambda x.e_1)\ e_2\ e_3$$
$$(\lambda x.(e_1\ e_2))\ e_3$$
$$\lambda x.(e_1\ e_2\ e_3)$$ ✓

$$(e_1\ (\lambda x.(e_2\ e_3\ e_4))\ e_5$$

$$\lambda x\,y.\ f\ (\lambda z.z\ z)\ (\lambda x.x)$$

$$\lambda x.(\lambda y.((f\ (\lambda z.(z\ z))\ (\lambda x.x))$$



Evalvacija izrazov

β-pravilo: $(\lambda x.e)\,e' = e[e'/x]$

redex

$(\lambda x.\ \underbrace{e}\ )\ e'$

$(\lambda x.\ (\lambda y.\ y^2+7)\ (x+8))\ (2+a)$

$(\lambda y.\ \cdots e \cdots)\ (e')$

$= (\lambda y.\ y^2+7)\,((2+a)+8) = ((2+a)+8)^2+7$

$\parallel$

$= (\lambda x.\ (x+8)^2+7\ )\,(2+a) = ((2+a)+8)^2+7$

Konfluenca: vsi načini računanja vodijo do istega rezultata.

Strategiji računanja:

$(\lambda x.e_1)\ ((\lambda y.e_2)\,e_3)$ $\qquad f(g(a))$

LENA (LAZY) ↙ $\qquad$ NEUČAKANA (EAGER) ↘

$e_1[(\lambda y.e_2)e_3/x]$ $\qquad (\lambda x.e_1)\ (e_2[e_3/y])$

↘

$e_1[e_2[e_3/y]/x]$

Programiramo v λ-računu

identiteta $\lambda x . x$

Boolove vrednosti in pogojni stavek:

true, false, if

if true A B = A

if false A B = B

$$\text{true} := \lambda x . \lambda y . x = \lambda x y . x$$

$$\text{false} := \lambda x . \lambda y . y = \lambda x y . y$$

$$\text{if} := \lambda b u v . b u v$$

if true A B =

$(\lambda b u v . b u v)$ true A B = true A B

$= (\lambda x y . x)$ A B

$= A$

Števila:

Churchevi numerali

$$\overline{3} := \lambda f . \lambda x . f(f(f x))$$

$$\overline{4} := \lambda f . \lambda x . f(f(f(f x)))$$

$\overline{1} := \lambda f.\lambda x.\ f\,x$

$\overline{0} := \lambda f.\lambda x.\ x$

$\overline{n}\ g\ a$ — g uporabi n-krat na a — $\underbrace{f(f\cdots fx)}_{n}$

Naslednik: $succ := \lambda n.\lambda f.\lambda x.\ f(n\,f\,x)$

Vsota: $plus := \lambda n\,m.\lambda f.\lambda x.\ n\,f\,(m\,f\,x)$

$f(f\cdots f(f(f(\cdots fx))))$ — n times, then m times

$\lambda n\,m.\lambda f\,x.\ n\,(m\,f)\,x$

$(mf)((mf)(\cdots(mf)\,x)\cdots)$ — n times

$true := \lambda x\,y.\ x$

$false := \lambda x\,y.\ y$

$if := \lambda b\,x\,y.\ b\,x\,y$

and true true = true

and true false = false

⋮

$p \wedge q$

---

if p then (if q then true else false) else false

---

if p then q else false

```
if (q) {
    return true;
} else {
  return false;
}
```

```
return q;
```