

Logično programiranje

Hornove formule

Logične formule:

→ izjavni račun: $\perp, \top, \wedge, \vee, \Rightarrow, \neg$ } logika 1. reda
→ predikatni račun: dodamo \forall in \exists } ali predikatni račun

→ primitivni predikati in relacije:

- enakost $a=b$
- teorija množic: $x \in A$
- geometrija: točka leži na premici, ...

Hornove formule:

$$\forall x_1, x_2, \dots, x_n. (\underbrace{\varphi_1 \wedge \varphi_2 \wedge \dots \wedge \varphi_m}_{\text{hipoteza}} \Rightarrow \underbrace{\psi}_{\text{sklep}})$$

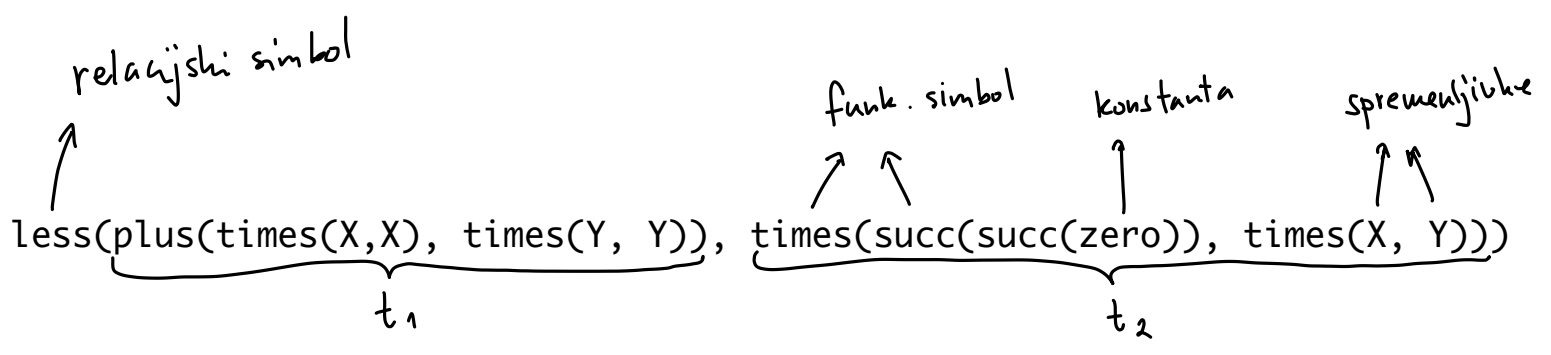
φ_i in ψ so osnovne formule

$$p(t_1, \dots, t_k)$$

p - relacijski simbol

t_j - termi/izrazi

Termi so zgrajeni iz spremenljivk in funkcijskih simbolov.



$$\forall x_1, \dots, x_n. \Psi$$

(ni hipotez) - dejstvo

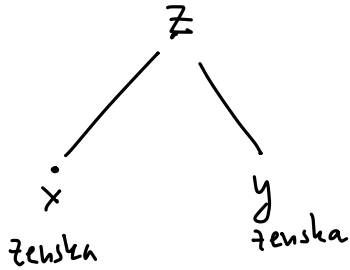
$$\varphi_1 \wedge \dots \wedge \varphi_n \Rightarrow \Psi$$

(ni spremenljivk)

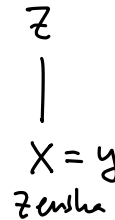
"x in y sta sestri ali
polsestri."

Primer:

$$\forall x y z. \text{otrok}(x, z) \wedge \text{otrok}(y, z) \wedge \text{zenska}(x) \wedge \text{zenska}(y) \Rightarrow \text{sestra}(x, y).$$



Lahko se zgodi: $x = y$



Funkcijske relacija:

$$n + 0 = n$$

$$n + \text{succ } m = \text{succ } (n + m)$$

Kako tako definicijo zapišemo s Hornovimi formulami?

Ideja:

dejstvo $a + b = c$ predstavimo z relacijo vsota (a, b, c) .

Primer:

$$5 + x = 6 \quad \dots \quad \text{vsota}(5, x, 6)$$

$$(x+1) + 2 = x+3 \quad \dots \quad \text{vsota}(\underbrace{x+1}_y, 2, \underbrace{x+3}_z)$$

$$\dots \quad \text{vsota}(x, 1, y) \wedge$$

$$x+1 = y$$

$$\text{vsota}(x, 3, z) \wedge$$

$$x+3 = z$$

$$\text{vsota}(y, 2, z)$$

$$y+2 = z$$

Funkcija f lahko predstavimo z relacijo ("graf funkcije f "):

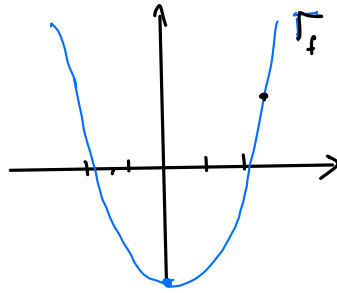
$$f: A \rightarrow B$$

$$\text{graf } \Gamma_f \subseteq A \times B$$

$$\Gamma_f = \{ (a, b) \mid f(a) = b \}$$

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

$$f: x \mapsto x^2 - 4$$



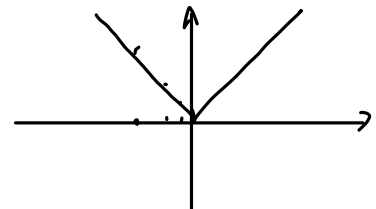
Primeri:

$$\Gamma_f \subseteq \mathbb{R} \times \mathbb{R}$$

$$\Gamma_f(x, y) \Leftrightarrow x = y \quad \text{to je funkcija}$$

$$x \mapsto x$$

$$\Gamma_f(x, y) \Leftrightarrow (x < 0 \wedge y = -x) \vee (x \geq 0 \wedge y = x)$$



$$f: x \mapsto |x|$$

Seštevanje naravnih števil:

$$n + 0 = n$$

$$\forall n. \text{ vsota}(n, \text{zero}, n)$$

$$n + \text{succ } m = \text{succ}(n+m)$$

$$\forall n, m. \text{ vsota}(n, \text{succ } m, \text{succ}(n+m))$$

$$\forall n, m, k. \text{ vsota}(n, m, k) \Rightarrow \text{vsota}(n, \text{succ } m, \text{succ } k)$$

Opazimo:

$$\forall x. \underbrace{\varphi_1 \vee \varphi_2 \Rightarrow \psi}_{(\varphi_1 \Rightarrow \psi) \wedge (\varphi_2 \Rightarrow \psi)}$$

ni Hornova

Nadomestimo z dvema H. formulama:

$$\forall x. \varphi_1 \Rightarrow \psi$$

$$\forall x. \varphi_2 \Rightarrow \psi$$

Formula $\forall x. \varphi_1 \wedge \varphi_2 \Rightarrow \psi_1 \wedge \psi_2$
je ekvivalentna dvema H. formulama

$$\forall x. \varphi_1 \wedge \varphi_2 \Rightarrow \psi_1$$

$$\forall x. \varphi_1 \wedge \varphi_2 \Rightarrow \psi_2$$

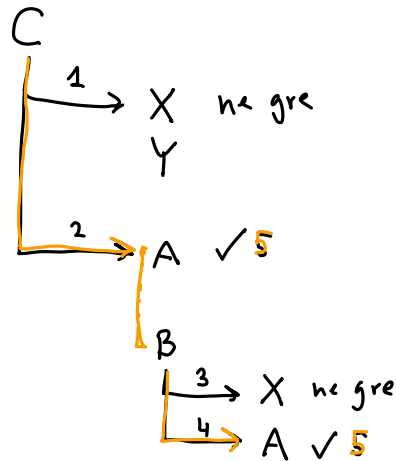
1. $X \wedge Y \Rightarrow C$

2. $A \wedge B \Rightarrow C$

3. $X \Rightarrow B$

4. $A \Rightarrow B$

5. A

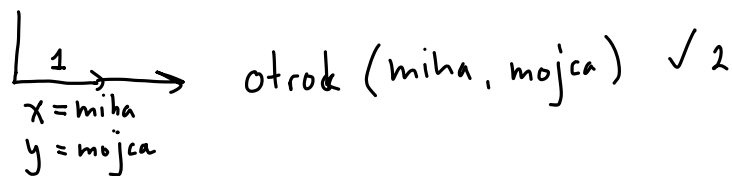


sledi C ?

1. $\forall x y. \text{otrok}(x, y) \Rightarrow \text{mlajsi}(x, y)$

2. $\text{otrok}(\text{miha}, \text{mojca})$

mlajsi (miha, mojca)



1. $\forall x. \text{sodo}(x) \Rightarrow \text{liho}(\text{succ}(x))$

2. $\forall y. \text{liho}(y) \Rightarrow \text{sodo}(\text{succ}(y))$

3. $\text{sodo}(\text{zero})$

sledi $\text{sodo}(\text{succ}(\text{succ}(\text{zero})))$? To

$\text{sodo}(\text{succ}(\text{succ}(\text{zero})))$

$\text{succ}(\text{succ}(\text{zero})) = \text{succ}(y)$
 $\text{succ}(\text{zero}) = y$



$\text{liho}(\text{succ}(\text{zero}))$

$\text{sodo}(\text{zero})$

$\text{sodo}(\text{zero})$

