

# Logično programiranje

## Hornove formule

Logische Formule:

- izjavní račun:  $\perp, T, \wedge, \vee, \Rightarrow, \neg$

→ predikativní račun: dodáme  $\forall$  in  $\exists$

→ primitivní predikati in relacije:

  - enakost  $a = b$
  - teorija množic:  $x \in A$
  - geometrija: točkaleži na premici, ...

logika I. reči  
ali  
predikativní račun

## Hornove formulē:

$\forall x_1, x_2, \dots, x_n . (\varphi_1 \wedge \varphi_2 \wedge \dots \wedge \varphi_m \Rightarrow \psi)$

$\varphi_i$  in  $\Psi$  so osnovne formule

$$p(t_1, \dots, t_k)$$

p - relacjiski simbol

$t_j$  - termi / i razzi

Termi so zgrajeni iz spremenljivk in funkcijskih simbolov.

The diagram shows a term in a logic programming language with the following components:

- relacijski simbol**: points to the first argument of the `less` predicate.
- funk. simbol**: points to the second argument of the `less` predicate.
- konstanta**: points to the third argument of the `less` predicate.
- spremenljivke**: points to the fourth argument of the `less` predicate.
- $t_1$** : groups the first two arguments (`plus(times(X,X), times(Y, Y))`).
- $t_2$** : groups the last two arguments (`times(succ(succ(zero))), times(X, Y))`).

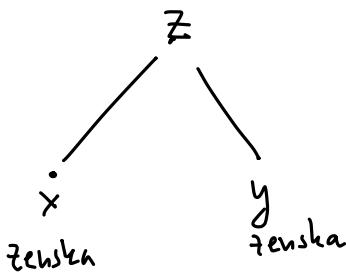
$\forall x_1, \dots, x_n. \Psi$  (ni hipotet) - dejstvo

$\varphi_1 \wedge \dots \wedge \varphi_n \Rightarrow \Psi$  (ni spremamljivk)

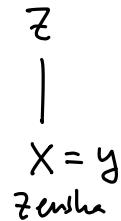
" $x$  in  $y$  sta sestri ali  
polsestri."

Primer:

$\forall x y z. \text{otrok}(x, z) \wedge \text{otrok}(y, z) \wedge \text{zenska}(x) \wedge \text{zenska}(y) \Rightarrow \text{sestra}(x, y).$



Lahko se zgodi  $x = y$



Funkcijске relacije:

$$n + 0 = n$$

$$n + \text{succ } m = \text{succ } (n + m)$$

Kako tako definicijo zapisemo s Hornovimi formulami?

Ideja:

dejstvo  $a + b = c$  predstavimo v relacijo vsota( $a, b, c$ ).

Primer:

$$5 + x = 6 \quad \dots \quad \text{vsota}(5, x, 6)$$

$$(x+1)+2=x+3 \quad \dots \quad \text{vsota}(\underbrace{x+1}_y, 2, \underbrace{x+3}_z)$$

$$\dots \quad \begin{array}{l} \text{vsota}(x, 1, y) \wedge \\ \text{vsota}(x, 3, z) \wedge \\ \text{vsota}(y, 2, z) \end{array} \quad \begin{array}{l} x+1=y \\ x+3=z \\ y+2=z \end{array}$$

Funkcijo  $f$  lahko predstavimo z relacijo ("graf funkcije  $f$ "):

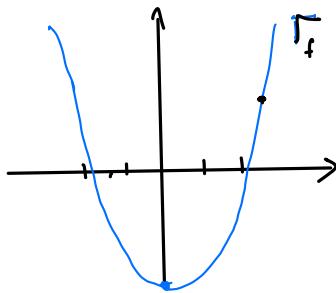
$$f : A \rightarrow B$$

$$\text{graf } \Gamma_f \subseteq A \times B$$

$$\Gamma_f = \{(a, b) \mid f(a) = b\}$$

$$f : \mathbb{R} \rightarrow \mathbb{R}$$

$$f : x \mapsto x^2 - 4$$

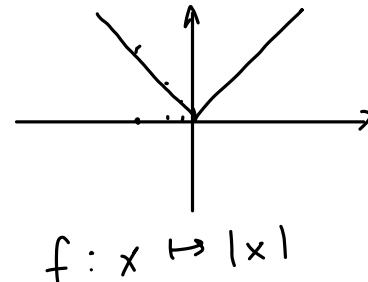


Primer:

$$\Gamma_f \subseteq \mathbb{R} \times \mathbb{R}$$

$$\Gamma_f(x, y) \Leftrightarrow x = y \quad \begin{matrix} \text{to je funkcija} \\ x \mapsto x \end{matrix}$$

$$\Gamma_f(x, y) \Leftrightarrow (x < 0 \wedge y = -x) \vee (x \geq 0 \wedge y = x)$$



Sestevanje naravnih števil:

$$n + 0 = n$$

$$\underline{\forall n. \text{vsota}(n, \text{zero}, n)}$$

$$n + \text{succ } m = \text{succ}(n+m)$$

$$\forall n, m. \text{vsota}(n, \text{succ } m, \frac{\text{succ}(n+m)}{k})$$

$$\underline{\forall n, m, k. \text{vsota}(n, m, k) \Rightarrow \text{vsota}(n, \text{succ } m, \text{succ } k)}$$

Opatimo:

$$\forall x. \underbrace{\varphi_1 \vee \varphi_2 \Rightarrow \psi}_{\Downarrow} \quad \text{ni Hormova}$$

$$(\varphi_1 \Rightarrow \psi) \wedge (\varphi_2 \Rightarrow \psi)$$

Nadomestimo z dvema tl. formulama:

$$\forall x. \varphi_1 \Rightarrow \psi$$

$$\forall x. \varphi_2 \Rightarrow \psi$$

Formula  $\forall x. \varphi_1 \wedge \varphi_2 \Rightarrow \psi_1 \wedge \psi_2$   
je ekvivalentna dvojna H. formulama

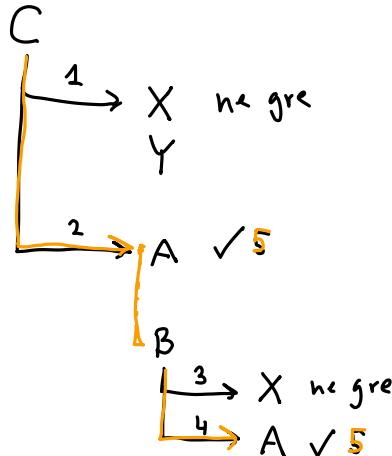
$$\forall x. \varphi_1 \wedge \varphi_2 \Rightarrow \psi_1$$

$$\forall x. \varphi_1 \wedge \varphi_2 \Rightarrow \psi_2$$


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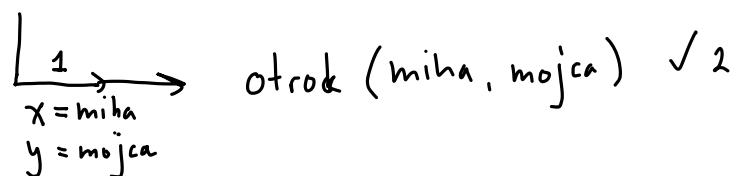
1.  $X \wedge Y \Rightarrow C$
2.  $A \wedge B \Rightarrow C$
3.  $X \Rightarrow B$
4.  $A \Rightarrow B$
5.  $A$

sledi  $C$ ?



1.  $\forall x y. \text{otrok}(x, y) \Rightarrow \underline{\text{mlajsi}}(x, y)$
2.  $\text{otrok}(\text{miha}, \text{mojca})$

mlajsi (miha, mojca)



1.  $\forall x. \text{sodo}(x) \Rightarrow \text{liho}(\text{succ}(x))$
2.  $\forall y. \text{liho}(y) \Rightarrow \text{sodo}(\text{succ}(y))$
3.  $\text{sodo}(\text{zero})$

sledi  $\text{sodo}(\text{succ}(\text{succ}(\text{zero})))$ ? To

