

Izpeljava tipov

Vaje

$\text{fun } \underset{\alpha}{x} \rightarrow \text{fun } \underset{\beta}{y} \rightarrow (\underset{\alpha * \beta * \beta}{x, y, y})$

$\underbrace{\quad}_{\alpha \rightarrow \beta} \rightarrow \alpha * \beta * \beta$

$\alpha \rightarrow \beta \rightarrow \alpha * \beta * \beta$

$\text{fun } \underset{\cancel{\alpha}}{f} \rightarrow \text{fun } \underset{\cancel{\beta}}{g} \rightarrow f(g \underset{\beta}{42})$

$\underbrace{\quad}_{\cancel{\alpha} \rightarrow \delta} \rightarrow \cancel{\beta} \rightarrow \underset{\delta}{\text{int} \rightarrow r}$

$\begin{array}{c} \tau_1 = \tau_2 \rightarrow \gamma \\ \tau_1 \uparrow \qquad \tau_2 \uparrow \\ h \qquad e \end{array}$

Rezultat:
 $\beta \mapsto \text{int} \rightarrow r$
 $\alpha \mapsto r \rightarrow \delta$

$(r \rightarrow \delta) \rightarrow (\text{int} \rightarrow r) \rightarrow \delta$

Odg: $(\alpha \rightarrow \beta) \rightarrow (\text{int} \rightarrow \alpha) \rightarrow \beta$

$\underbrace{\quad}_{\cancel{\alpha} \rightarrow \delta} \rightarrow \cancel{\beta} \rightarrow r$
 $f(g \underset{\beta}{42})$

$\begin{array}{c} g \underset{\beta}{42} \underset{\text{int}}{\sim} \\ \beta \underset{\text{int} \rightarrow r}{\sim} r \end{array}$

if e_1 then e_2 else e_3
 $\frac{e_1 \in T_1 \quad e_2 \in T_2}{T_1 = T_2}$

$$\text{if } \underbrace{3 < 5}_{\text{int}} \text{ then } (\underbrace{\text{fun } x \rightarrow x}_{\alpha \rightarrow \alpha}) \text{ else } (\underbrace{\text{fun } y \rightarrow (y, y)}_{\beta \rightarrow \beta * \beta})$$

$$\alpha \rightarrow \alpha \equiv \beta \rightarrow \beta * \beta \quad \checkmark$$

$$\alpha \equiv \beta \quad \checkmark$$

$$\alpha \equiv \beta * \beta \quad \checkmark$$

$$\alpha \equiv \alpha * \alpha \quad \text{NOT RESITIVE.}$$

$$((\cdot), \cdot) \quad ((\cdot), \cdot)$$

Resitzer:
 $\alpha \mapsto \beta$

$$3 < 5$$

$$3.14 < 5 \quad 3.1 < 5.7$$

$$3.1 + 5 <$$

1. možnost (OCaml)

$+ : \text{int} \rightarrow \text{int} \rightarrow \text{int}$ (globální tip $\approx +$)
 $+ : \text{float} \rightarrow \text{float} \rightarrow \text{float}$

2. možnost (SML)

Polimorfismus (ad-hoc)

$+ : \text{int} \rightarrow \text{int} \rightarrow \text{int}$
 $+ : \text{float} \rightarrow \text{float} \rightarrow \text{float}$

} dva tipy,
 noben v globálním

Jawa má ad-hoc polimorfismus (overloading)

```

class Foo {
    :
    public int f(int x) { ... }           int → int
    public bool f(float x, float y) { ... } float × float → bool
}

```

$$< : \alpha \rightarrow \alpha \rightarrow \text{bool}$$

Naloga 1:

$$\text{fun } h \rightarrow \text{fun } f \rightarrow h \underbrace{(f \ 0)}_{\alpha} \underbrace{1}_{\text{int}}$$

Naloga 2:

$$\text{fun } g \ x \rightarrow g(g(x + 3))$$

Naloga 1 - rešitev

$$(\cancel{(\text{int} \rightarrow \beta)} \rightarrow \text{int} \rightarrow \alpha) \rightarrow (\text{int} \rightarrow \beta) \rightarrow \alpha \quad (\text{Andrej})$$

$$(\alpha \rightarrow (\text{int} \rightarrow \beta)) \rightarrow ((\text{int} \rightarrow \alpha) \rightarrow \beta) \quad (\text{Gregor})$$

$$(\alpha \rightarrow (\text{int} \rightarrow \beta)) \rightarrow ((\text{int} \rightarrow \alpha) \rightarrow \beta) \quad (\text{Drejč})$$

Naloga 1 - postopek:

$$\begin{array}{c}
 \text{fun } h \rightarrow \text{fun } f \\
 \cancel{x} \\
 (\cancel{x} \rightarrow \delta) \\
 (x \rightarrow (\text{int} \rightarrow \varepsilon)) \\
 \vdots \\
 (\beta \rightarrow \text{int} \rightarrow \varepsilon) \rightarrow (\text{int} \rightarrow \gamma) \rightarrow \varepsilon \\
 (\alpha \rightarrow \text{int} \rightarrow \beta) \rightarrow (\text{int} \rightarrow \alpha) \rightarrow \beta \quad \text{odgovor}
 \end{array}
 \rightarrow
 \begin{array}{c}
 \delta = \text{int} \rightarrow \varepsilon \\
 \underbrace{\gamma}_{\gamma \rightarrow \delta} \\
 \underbrace{(h \ (f \ 0))}_{\beta = \text{int} \rightarrow \gamma} \quad \underbrace{1}_{\text{int}} \\
 \varepsilon
 \end{array}$$

Razitev:

$$\beta = \text{int} \rightarrow \gamma$$

$$\alpha = \gamma \rightarrow \delta$$

$$\delta = \text{int} \rightarrow \varepsilon$$

$$\begin{array}{c}
 \delta = \text{int} \rightarrow \varepsilon \\
 \underbrace{(h \ (f \ 0))}_{\beta = \text{int} \rightarrow \gamma} \quad \underbrace{1}_{\text{int}} \\
 \underbrace{\gamma}_{\gamma \rightarrow \delta} \\
 \underbrace{\delta}_{\text{int}}
 \end{array}$$

$$\begin{array}{c}
 \text{apple} \\
 \text{X} \\
 \text{banana} \\
 \text{Y} \\
 \text{chaitin: } X = Y \rightarrow \alpha
 \end{array}$$

Naloga 2 - postopek:

$$\begin{array}{c}
 \text{fun } g \rightarrow \text{fun } x \\
 \cancel{x} \\
 \text{int} \rightarrow \gamma \\
 \text{int} \rightarrow \text{int}
 \end{array}
 \rightarrow
 \begin{array}{c}
 \text{fun } x \rightarrow g \\
 \cancel{\beta} \\
 \text{int} \rightarrow \gamma
 \end{array}$$

$$\begin{array}{c}
 \delta = \text{int} \rightarrow \varepsilon \\
 \underbrace{(g \ (g \ (x + 3)))}_{\alpha = \text{int} \rightarrow \gamma} \\
 \underbrace{\alpha}_{\alpha = \text{int} \rightarrow \gamma} \\
 \underbrace{\gamma}_{\gamma \rightarrow \delta} \\
 \underbrace{x + 3}_{\beta = \text{int} \rightarrow \gamma} \\
 \underbrace{\beta}_{\beta = \text{int} \rightarrow \gamma} \\
 \underbrace{\varepsilon}_{\delta = \text{int}}
 \end{array}$$

$$\begin{array}{c}
 \text{Razitev:} \\
 \beta = \text{int} \\
 \alpha = \text{int} \rightarrow \gamma \\
 \gamma = \text{int} \\
 \delta = \text{int}
 \end{array}$$

$$\begin{array}{c}
 (\text{int} \rightarrow \text{int}) \rightarrow (\text{int} \rightarrow \text{int})
 \end{array}$$

$$\begin{array}{c}
 (\alpha \rightarrow \beta) \rightarrow (\alpha * \gamma \rightarrow \beta) \\
 \underbrace{\hspace{10em}}_{\beta} \\
 \text{fun } f^{\alpha \rightarrow \beta} \rightarrow (\text{fun } p^{\alpha * \gamma} \rightarrow f^{\alpha \rightarrow \beta} (f^{\text{fst}} p^{\alpha}, p^{\gamma}))
 \end{array}$$

Naloga:

```
let rec f x = (if x = 0 then 1 else x * f (x - 1))
```

A handwritten derivation of the recursive factorial function f . The derivation shows the type annotations for each part of the expression:

- let rec f =** (Type: $\text{fun } x \rightarrow$)
- if $x = 0$ then 1 else** (Type: $\beta = \text{bool}$)
- $x * f(x - 1)$** (Type: $\alpha = \text{int}$, $\beta = \text{int}$, $\gamma = \text{int}$)

Annotations below the expression show the types of the variables and functions:

- $\cancel{\text{int} \rightarrow r}$ (Type of x)
- $\cancel{\text{int} \rightarrow \text{int}}$ (Type of f)
- int (Type of x)
- β (Type of x)
- $\beta = \text{int}$ (Type of $=$)
- int (Type of 0)
- int (Type of 1)
- int (Type of $*$)
- int (Type of f)
- $\alpha = \text{int}$ (Type of f)
- $\beta = \text{int}$ (Type of f)
- $\gamma = \text{int}$ (Type of f)
- $\text{int} = \text{int}$ (Type of $=$)
- int (Type of $-$)
- int (Type of f)
- int (Type of f)
- int (Type of f)

On the right, the **Rasitw:** (Results) are listed:

- $\beta \mapsto \text{int}$
- $\alpha \mapsto (\text{int} \rightarrow r)$
- $f \mapsto \text{int}$

Na hohem: $\text{lev}_n = \text{dern}'$

$$(\text{int} \rightarrow \text{int}) = (\text{int} \rightarrow \text{int}) \quad \checkmark$$

Odgovor: $f : \text{int} \rightarrow \text{int}$

Naloga: Izpelji tip f

Let rec $f\ b\ x = \text{if } b \text{ then } (f\ x\ x) \text{ else } (f\ x\ b)$

$f \equiv \text{fun } b \rightarrow (\text{fun } x \rightarrow \text{if } b \text{ then } (f \ x \ x) \text{ else } (f \ x \ b))$

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$f \equiv \text{fun } b \rightarrow (\text{fun } x \rightarrow \text{if } b \text{ then } ((f x) x) \text{ else } ((f x) b))$

\cancel{x}
 $\cancel{x \rightarrow \delta}$
 $\cancel{r \rightarrow (r \rightarrow \varepsilon)}$
 $\text{bool} \rightarrow (\text{bool} \rightarrow \varepsilon)$

\cancel{x}
 \vdots
 \vdots
 \vdots
 \cancel{x}
 \vdots
 \vdots
 bool

$\alpha = r \rightarrow \delta$
 $\delta = r \rightarrow \varepsilon$
 $r \rightarrow \varepsilon$
 ε
 Resitzer:
 $\alpha \mapsto r \rightarrow \delta$
 $\delta \mapsto r \rightarrow \varepsilon$
 $r \mapsto \text{bool}$

$$\text{bool} \rightarrow (\text{bool} \rightarrow \varepsilon) \equiv \text{bool} \rightarrow (\text{bool} \rightarrow \varepsilon) \quad \checkmark$$

$$f : \text{bool} \rightarrow \text{bool} \rightarrow \epsilon$$