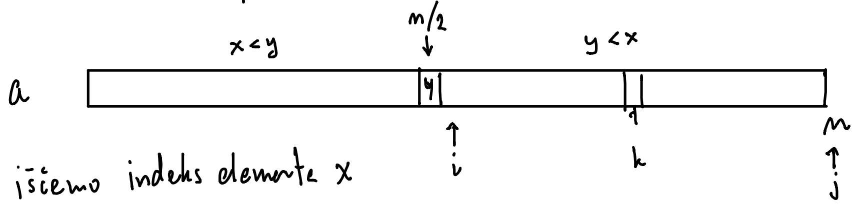
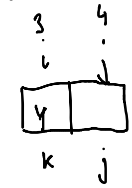
Bisekcija:

urejena tabela



iščemo indeks elementa x



Urejanje tabel

Naloga: dano tabelo elementov uredi glede na dano ureditev.

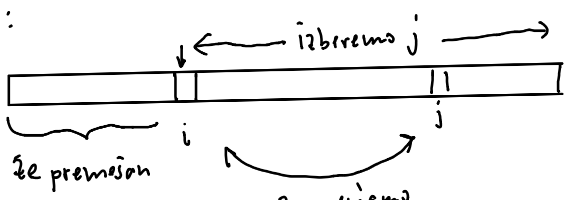
Vaja: Kako enakomerno premešamo tabelo?

$$a = [a_0, \ldots, a_{n-1}]$$

$\downarrow$ naključne permutacije π na $\{0, \ldots, n-1\}$

$$[a_{\pi(0)}, \ldots, a_{\pi(n-1)}]$$

Postopek:



$$\frac{1}{n} \qquad \frac{\cancel{n-1}}{n} \cdot \frac{1}{\cancel{n-1}} = \frac{1}{n}$$

Dve vrsti urejanja:

→ urejanje na mestu: tabelo a spremenimo, da je urejena
(in place)

→ urejanje, ki prvotne tabele ne spremeni, vrne novo urejeno tabelo

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Urejanje na mestu, prvi poskus:

0, 1, 2, 3, …, n−2, n−1

3, 1, 5, 8, 2, 4, 7, 6

i, j

1, 3, 5, 8, 2, 4, 7, 6

i, j

1, 2, 5, 8, 3, 4, 7, 6

i, j

⋮

8, 7, 6

i

7, 8

n−2

i

8

To je <u>urejanje z izbiranjem</u>:

vhod: tabela $a = [a_0, \dots, a_{n-1}]$, $n = \text{len}(a)$

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for i in range(0, n-1):
    k = indeks najmanjšega elementa v podtabeli
        a[i], ..., a[n-1]

    zamenjaj a[i] in a[k]
```

Časovna zahtevnost: **koraki**

$i = 0$: $j = 1, \dots, n-1$: $n-1$

$i = 1$: $j = 2, \dots, n-1$: $n-2$

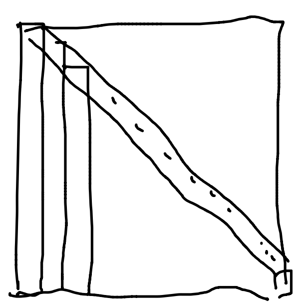
$i = 2$: $j = 3, \dots, n-1$: $n-3$

$\vdots$ $\vdots$

$i = n-2$ $j = n-1, \dots, n-1$ 1

$$1 + 2 + 3 + \dots + (n-1) = \frac{n(n-1)}{2} = \frac{n^2}{2} - \frac{n}{2}$$

$$= O(n^2)$$



Urejanje z zlivanjem

Zlivanje: Imamo urejeni tabeli a in b.
Združimo ju v skupno tabelo, ki mora biti urejena.

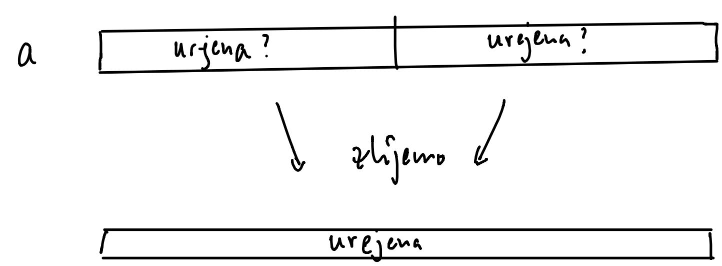
$$a = [1, 4, 8, 10, 20, 22] \quad (m) \qquad b = [2, 3, 7, 15, 16] \quad (n)$$

$$\uparrow \ \uparrow \ \underset{i}{\uparrow} \qquad\qquad \uparrow \ \uparrow \ \uparrow \ \underset{j}{\uparrow}$$

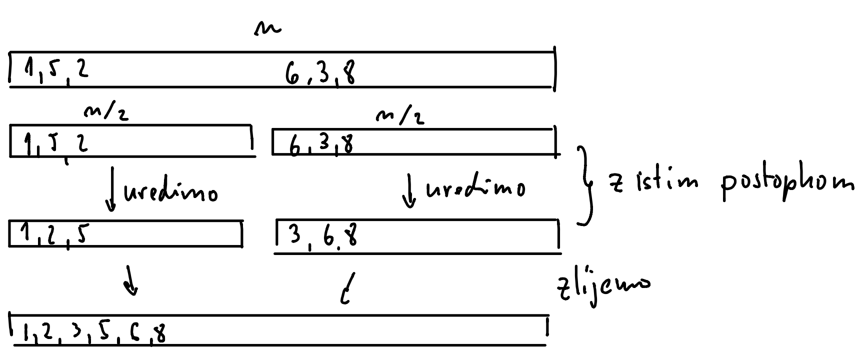
$$c = [1, 2, 3, 4, 7, \overset{i+j}{\overset{\downarrow}{8}} \qquad] \quad (m+n)$$

Časovna zahtevnost: $O(m+n)$

Urejanje z zlivanjem:



Algoritem:

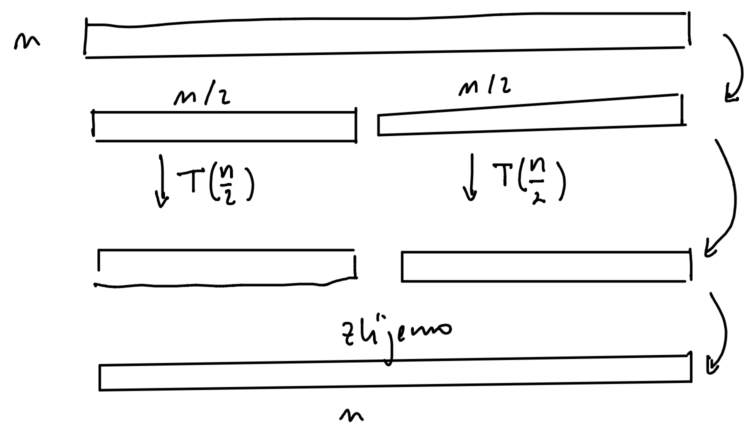


Časovna zahtevnost:

$T(n) :=$ število korakov za urejanje z zlivanjem tabele dolžine n

Knjigovodstvo

koraki



$c_1 \cdot n$

$\frac{n}{2} + \frac{n}{2} = n$

$T(\frac{n}{2}) + T(\frac{n}{2}) = 2 \cdot T(\frac{n}{2})$

$c_2 \cdot n$

n

$$T(n) = n + 2 \cdot T\left(\frac{n}{2}\right) + n = 2 \cdot T\left(\frac{n}{2}\right) + 2n$$

$(c_1 + c_2) \cdot n$

$$2 \cdot T\left(\frac{n}{2}\right) + c \cdot n$$

$c = 1$

Dobimo:

$$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + n \quad , \quad T(0) = 1, \; T(1) = 1$$

$$T(n) = 2 \cdot \overbrace{\left(2 \cdot T\left(\frac{n}{4}\right) + \frac{n}{2}\right)}^{T(\frac{n}{2})} + n = 4 \cdot T\left(\frac{n}{4}\right) + 2n$$

$$= 4 \cdot \left(2 \cdot T\left(\frac{n}{8}\right) + \frac{n}{4}\right) + 2n = 8 \cdot T\left(\frac{n}{8}\right) + 3n$$

$$\vdots$$

$$= 2^3 \cdot T\left(\frac{n}{2^3}\right) + 3n$$

$= \quad \dots \quad 2^k \cdot T\left(\frac{n}{2^k}\right) + k \cdot n$

ustavi se pri $T(1)$ $\parallel$ $\frac{n}{2^k}$

$\frac{n}{2^k} = 1$

$= 2^{\log_2 n} \cdot T(1) + (\log_2 n) \cdot n$

$n = 2^k$

$= n \cdot T(1) + n \cdot \log_2 n$ $\Rightarrow k = \log_2 n$

$k = \log_2 n$

$= O(n \cdot \log_2 n)$

Urejanje z zlivanjem je v vseh primerih $O(n \cdot \log_2 n)$

$O(\log_a n) = O(\log_b n)$ ker

$$\log_a n = \frac{\log_b n}{\log_b a}$$