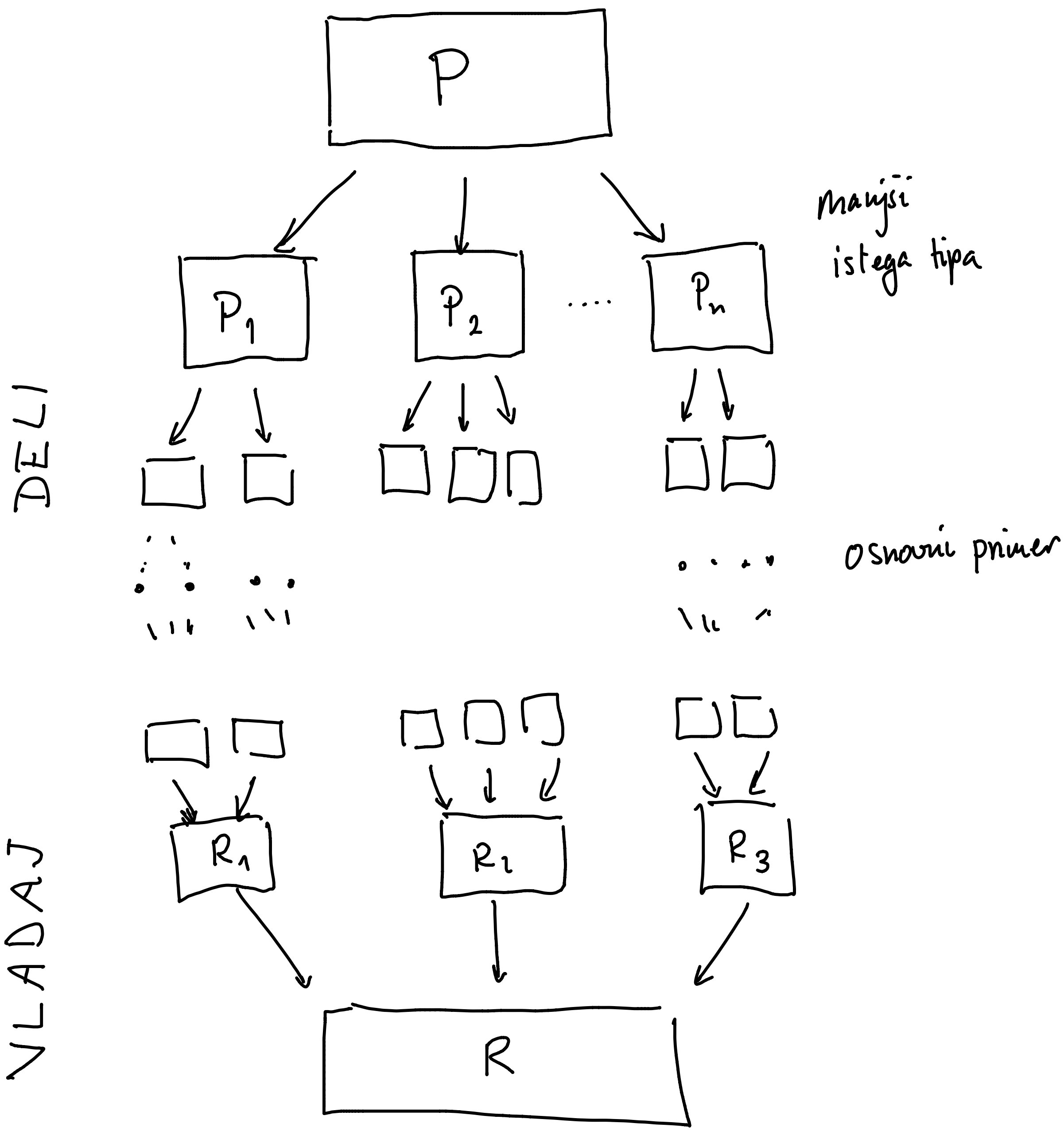
Deli & vladaj



Divide & conquer

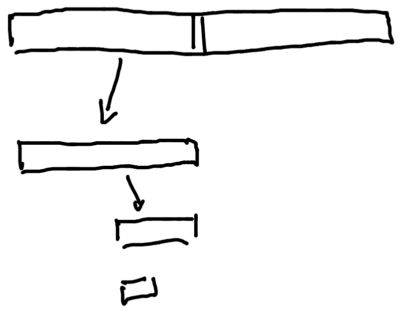
Primeri:

- Urejanje z zlivanjem
  - deli: tabelo razdelimo na dve podtabeli — 2 podproblema velikosti $\frac{n}{2}$
  - vladaj: urejeni tabeli zlijemo

- Bisekcija

a urejena

iščemo indeks x



deli: odloči se, katerega od obeh manjših podproblemov bomo obravnavali — 1 podproblem velikosti $\frac{n}{2}$

vladaj: ni dela

Časovna zahtevnost deli & vladaj:

Imamo algoritem deli & vladaj:

- k podproblemov velikosti $\frac{n}{c}$, $c > 1$
- osnovni primer $n \leq 1$, rešimo v 1 koraku
- faza deli zahteva $f(n)$ korakov
- faza vladaj zahteva $g(n)$ korakov

Število korakov:

$$T(0) = 1$$
$$T(1) = 1$$
$$T(n) = \underbrace{f(n)}_{\text{deli}} + \underbrace{k \cdot T\left(\frac{n}{c}\right)}_{\text{rekurzivni klici}} + \underbrace{g(n)}_{\text{vladaj}}$$

$$= h(n) + k\,T\left(\frac{n}{c}\right)$$, kjer $h(n) = f(n) + g(n)$

$$\Rightarrow \quad T(n) = h(n) + k \cdot T\left(\frac{n}{c}\right)$$
$$= h(n) + k \cdot \left(h\left(\frac{n}{c}\right) + k \cdot T\left(\frac{n}{c^2}\right)\right) =$$
$$= h(n) + k \cdot h\left(\frac{n}{c}\right) + k^2 \cdot T\left(\frac{n}{c^2}\right) =$$
$$= h(n) + k \cdot h\left(\frac{n}{c}\right) + k^2 \cdot \left(h\left(\frac{n}{c^2}\right) + k \cdot T\left(\frac{n}{c^3}\right)\right) =$$
$$= h(n) + k \cdot h\left(\frac{n}{c}\right) + k^2 \cdot h\left(\frac{n}{c^2}\right) + k^3 \cdot T\left(\frac{n}{c^3}\right) =$$
$$\vdots$$
$$\sum_{i=0}^{m-1} k^i \cdot h\left(\frac{n}{c^i}\right) + k^m \cdot T\left(\frac{n}{c^m}\right)$$ m-korakov

Ustavimo se pri $T(1)$, se pravi $\frac{n}{c^m} = 1 \Rightarrow m = \log_c n$

Vstavimo m:

$$T(n) = \sum_{i=0}^{\log_c n - 1} k^i h\left(\frac{n}{c^i}\right) + k^{\log_c n} \cdot T(1)$$

če postavimo $T(1) = h(1)$

$$= \sum_{i=0}^{\log_c n} k^i \cdot h\left(\frac{n}{c^i}\right)$$

Primer:

Urejanje z zlivanjem: $k=2$, $c=2$, $h(n) = n$

$$T(n) = \sum_{i=0}^{\log_2 n} 2^i \cdot \frac{n}{2^i} = \sum_{i=0}^{\log_2 n} n = n(\log_2 n + 1)$$

$$\in O(n \cdot \log_2 n)$$

Bisekcija: $k=1$, $c=2$, $h(n) = 1$

$$T(n) = \sum_{i=0}^{\log_2 n} 1^i \cdot 1 = \log_2 n + 1 \in O(\log_2 n)$$

Primer: $k=2$, $c=2$, $h(n) = n^2$

$$T(n) = \sum_{i=0}^{\log_2 n} 2^i \cdot \left(\frac{n}{2^i}\right)^2 = \sum_{i=0}^{\log_2 n} \frac{n^2}{2^i} =$$

$$= n^2 \cdot \sum_{i=0}^{\log_2 n} \frac{1}{2^i} \leq n^2 \cdot \sum_{i=0}^{\infty} \frac{1}{2^i} = 2n^2 \in O(n^2)$$

↳ je med 1 in 2

$$\sum_{i=0}^{r} \alpha^i = \frac{\alpha^{r+1} - 1}{\alpha - 1} \in O(\alpha^{r+1}$$

Strassenov algoritem:

$$k = 7, \quad c = 2, \quad h(n) = n$$

$$T(n) = \sum_{i=0}^{\log_2 n} 7^i \cdot \left(\frac{n}{2^i}\right)^2 = n^2 \cdot \sum_{i=0}^{\log_2 n} \left(\frac{7}{4}\right)^i$$

$$\leq n^2 \cdot \left(\frac{7}{4}\right)^{\log_2 n} \cdot \text{Const.}$$

$$\approx n^2 \cdot \left(\frac{7}{4}\right)^{\log_2 n} = n^2 \cdot \left(2^{\log_2 \frac{7}{4}}\right)^{\log_2 n}$$

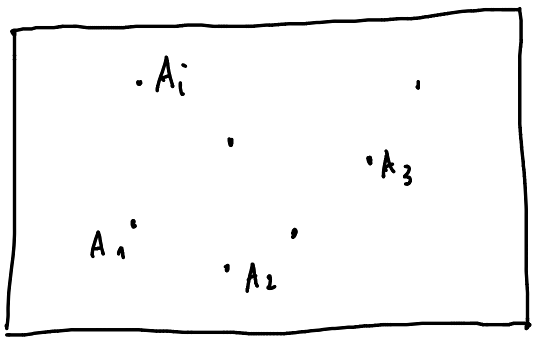
$$= n^2 \cdot \left(2^{\log_2 n}\right)^{\log_2 \frac{7}{4}} =$$

$$= n^2 \cdot n^{\log_2 \frac{7}{4}} =$$

$$= n^{2 + \log_2 \frac{7}{4}} = n^{\log_2 7} \in O(n^{2.81})$$

$$\log_2 7 \leq 2.81$$

Problem najbližjih točk:



Poišči par A_i, A_j, $i \neq j$
ki ima najmanjšo razdaljo
$d(A_i, A_j)$ minimalen

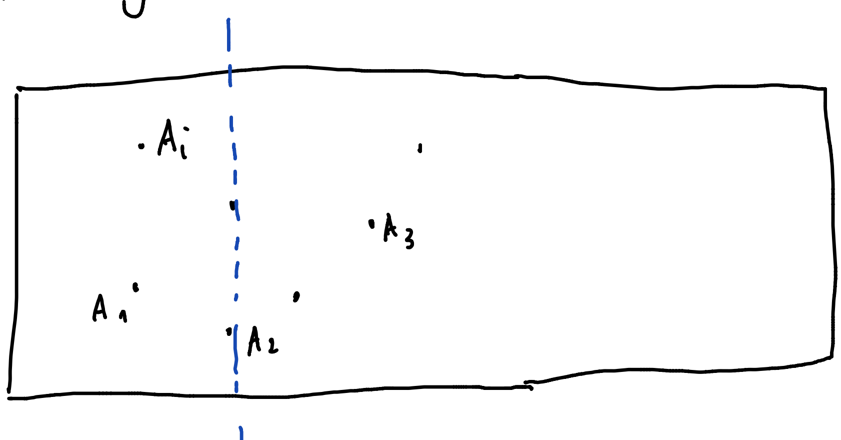
VHOD: $[A_1, \ldots, A_n]$ tabela $A_i = (x_i, y_i)$

IZHOD: Indeksa i, j

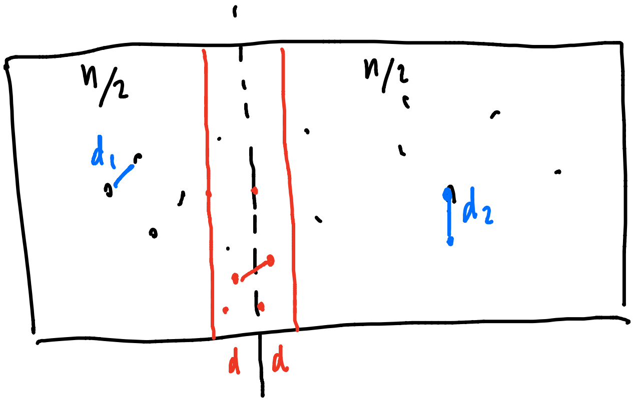
Naivna rešitev: obravnavaj vse pare (i,j), $i<j$,

$$O(n^2)$$

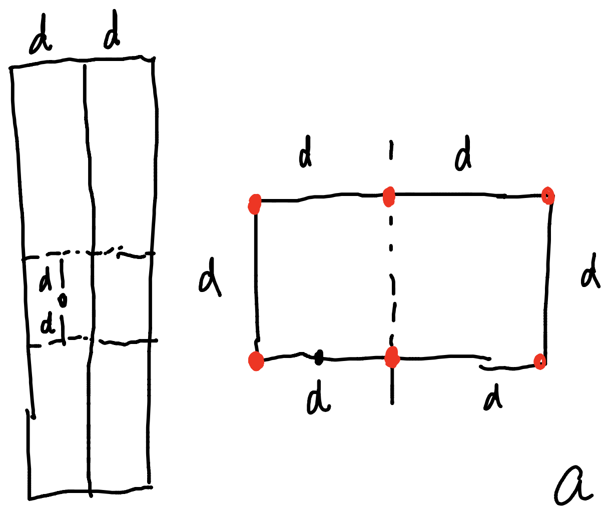
Deli & Vladaj



Točke razdelimo na pol glede na njihove x-koordinate



$d = \min(d_1, d_2)$



a

a_x

$k = 2, \quad c = 2,$

dobim točke v pasu

sort po y

$$h(n) = \underset{\text{deli}}{1} + \underbrace{\log_2 n + n\log n + 5n}_{n \cdot \log n}$$

vladaj

izboljšava: na začetku en sort po y

$$1 + n + 5n$$

da dobimo točke v pasu

pregledamo pas

$h(n) = n$

Odgovor: $\underbrace{n \cdot \log n}_{\text{sort po } x} + \underbrace{n \cdot \log n}_{\text{sort po } y} + \underbrace{n \cdot \log n}_{\text{deli \& vladaj}} \in O(n \log n)$