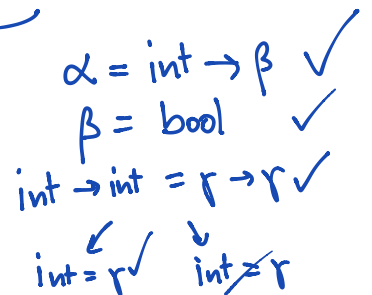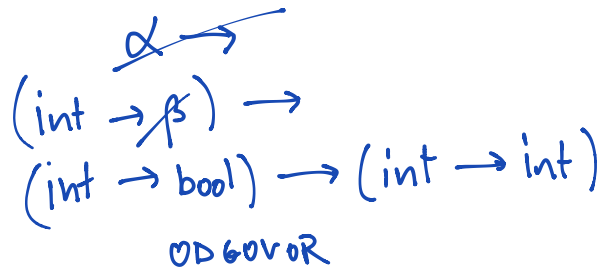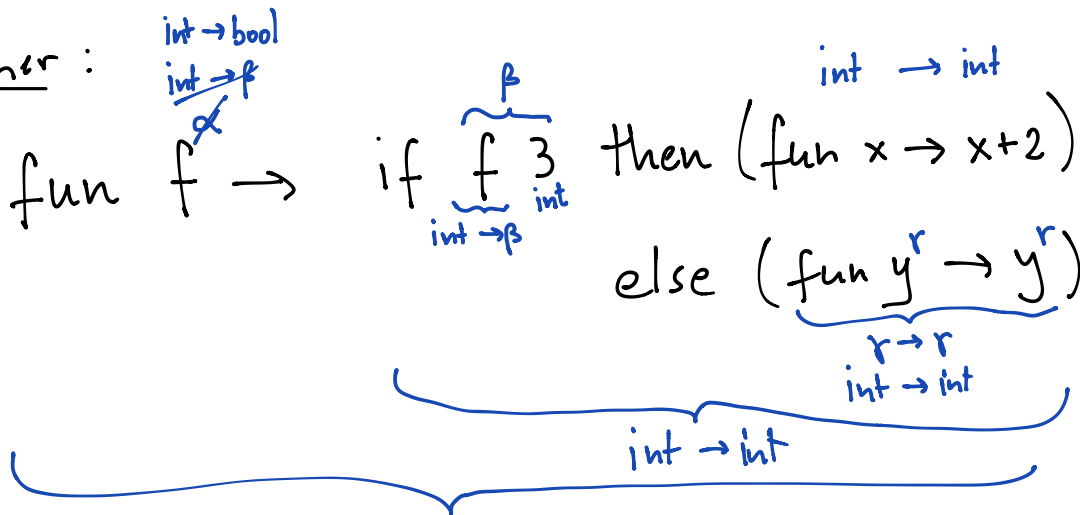# Izpeljava tipov – vaje

Ponovimo:

Naloga: izpelji glavni tip izraza e

① Izračunaj kandidata τ za tip e + enačbe

② Rešimo enačbe

Odgovor: glavni tip (τ z upoštevano rešitvijo enačb)

ali e nima tipa (ker enačbe nimajo rešitve)

Primer:

$$\text{fun } f \to \quad \text{if } \underbrace{\underset{int \to \beta}{f} \ \underset{int}{3}}_{} \text{ then } \left(\text{fun } x \to x+2\right)$$

$$\text{else } \left(\text{fun } y^r \to y^r\right)$$

over f: $\overset{int \to bool}{\underset{\alpha}{int \to f}}$

over $f\ 3$: β

over then: $int \to int$

over else: $\underbrace{y^r \to y^r}_{int \to int}$, $r \to r$

$\underbrace{}_{int \to int}$

$\alpha \to$

$(int \to \beta) \to$

$(int \to bool) \to (int \to int)$

ODGOVOR

$\alpha = int \to \beta$ ✓
$\beta = bool$ ✓
$int \to int = r \to r$ ✓
$int = r$ ✓ $\quad int = r$ ✓

Naloga 1:

$(a, b) : A \times B$
A   B $A * B$ Ocaml
          (A, B) Haskell

```
fun x -> (fun y -> (x, y, y))
```

(vsak dela sam do 8:40)

Razmisli:
$$a \to (b \to c)$$
$$(a \to b) \to c$$

① $\alpha \to \beta \to \alpha \times \beta \times \beta$ $\gg$

② $\alpha \to (\beta \to \alpha \times \beta \times \beta)$

Ali je to isto kot   fun x y → (x, y, y) ?

DA. Ohrajšava   fun p_1 p_2 ... p_n → e   za
$$\text{fun } p_1 \to \text{fun } p_2 \to \cdots \to \text{fun } p_n \to e$$

Naloga 2:

```
fun f -> fun g -> f (g 42)
```
$\overset{\beta \to \gamma}{}$ $\overset{int \to \beta}{}$

Rešujemo do 8:50

$\overset{\beta \to \gamma}{\nearrow}$ $\overset{int \to \beta}{\nearrow}$

$(\beta \to \gamma) \to (int \to \beta) \to \gamma$ $\overset{\gamma}{\downarrow}$ $\overset{}{\downarrow}$ bool

$\underbrace{g\ 42}_{\beta}$ $\underbrace{f(g\,42)}_{\gamma}$

$\alpha \rightharpoonup \beta \to \alpha$?

$(\alpha \to \beta) \to (int \to \alpha) \to \beta$

~~$(int \to \alpha) \to int \to \alpha$~~
          ?

Naloga 3

do 9:05

```
if 3 < 5 then (fun x -> x) else (fun y -> (y, y))
```
$\underbrace{\phantom{3<5}}_{bool}$ $\underbrace{}_{\alpha \to \alpha}$ $\underbrace{}_{\beta \to \beta \times \beta}$

$\checkmark$

)

~~unit~~ => ('a -> 'a * 'a)   ?   ↓

$$\alpha \to \alpha = \beta \to \beta \times \beta$$
$$\hookrightarrow \quad \alpha = \beta$$
$$\hookrightarrow \quad \alpha = \beta \times \beta$$

Rešitev
$$\alpha \mapsto \beta$$

$$\boxed{\beta = \beta \times \beta} \quad \text{NI REŠITVE}$$
$$\beta$$

$$\beta \to \beta$$

pri čemer je $\beta = \beta \times \beta$

$$(\beta \times \beta \text{ as } \beta) \to \beta$$

Naloga:

$$\overset{\alpha \quad \beta}{\text{let rec f x}} = \text{(if x = 0 then 1 else x * f (x - 1))}$$

$$\underset{\alpha}{\text{let rec f}} = \text{fun } \overset{\beta}{x} \to \underbrace{\text{(if x = 0 then 1 else x * f (x - 1))}}$$
$$t$$

Odgovor: t   pri pogoju $\alpha = t$

if x = 0 then $\underset{int}{\underline{0}}$ else $\underset{int}{\underline{1}}$

$\underbrace{\hspace{4cm}}_{int}$

if P then A else B   → istega tipa kot A,
pri pogoju, da imata
A in B isti tip
(in P ima tip bool)
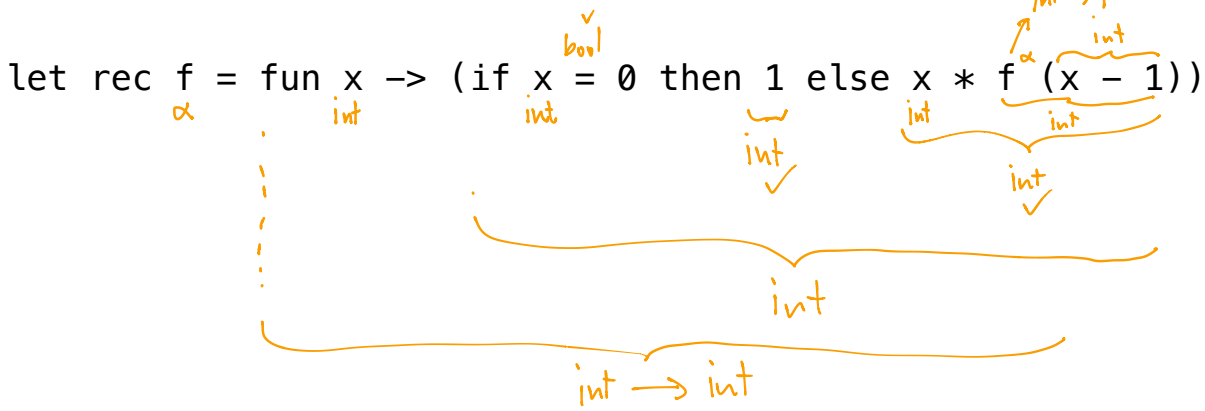
TO NI FUNKCIJA

~~funkc~~ if x = 0 then 0 else 1

$\boxed{\sin x} \in \mathbb{R}$

TO NI FUNKCIJA!!

Sin je funkcija
$\lambda x. \sin x$ je funkcija

$3x+5$ linearna "funkcija"

$x \mapsto 3x+5$

```
let rec f = fun x -> (if x = 0 then 1 else x * f (x - 1))
```
α int   bool int   int   int

with annotations: β, int $\to \beta$, α, int, int, int, int

int (over then branch)

int (large brace)

$int \to int$

$\alpha = int \to \beta$ $\beta = int$

$\alpha = int \to \beta$
$\beta = int$ ✓
$\alpha = int \to int$
zaradi rekurzije

ODGOVOR: $\underline{\underline{int \to int}}$ po enačbah

Rešitev:
$\beta \mapsto int$ ✓
$\alpha \mapsto int \to int$

Kako izpeljemo tip za

$$\text{match } \overset{\tau_1}{e_1} \text{ with}$$
$$| \; [] \to \overset{\tau_2}{e_2}$$
$$| \; \overset{\alpha}{x} :: \overset{\alpha \, list}{xs} \to \overset{\tau_3}{e_3}$$

τ_2 enačbe
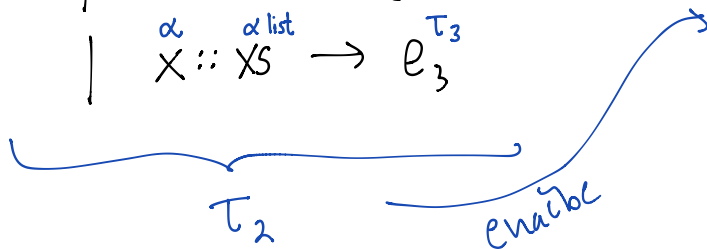
E_1
E_2
E_3

$\tau_2 = \tau_3$

$\tau_1 = \alpha \; list$

OPAZIMO

if P then A else B
‖
match P with
| true → A
| false → B

Naloga :

$$\text{fun } \ell^{\alpha} \to \text{match } \ell^{\alpha} \text{ with}$$
$$| \ [\ ] \to \underset{\text{int}}{0} :: [\ ] \quad \text{int list} \quad \Big] \text{ int list}$$
$$| \ \underset{\beta}{x} :: \underset{\beta \text{ list}}{xs} \to \underset{}{x} \quad \beta$$

$$\underbrace{\qquad\qquad\qquad\qquad\qquad}_{\alpha \to \text{int list}}$$

$$\alpha = \beta \text{ list} \ \bigg\}_{\sigma} \qquad \overset{\gamma}{(\text{int list}) \text{ list}} \to \text{int list}$$
$$\text{int list} = \beta$$
$$\alpha = (\text{int list}) \text{ list}$$

Naloga :

$$\text{match } \underset{\beta \text{ list}}{\overset{①}{[\ ]}} \text{ with}$$
$$| \ [\ ] \to \underset{\gamma \text{ list}}{\overset{②}{[\ ]}}$$
$$| \ \underset{\alpha}{x} :: \underset{\alpha \text{ list}}{xs} \to \underset{\overset{③}{\ } \alpha \text{ list}}{xs}$$

$$\underbrace{\qquad\qquad\qquad}$$

$$\boxed{② \ \gamma \text{ list}}$$

$$① = \alpha \text{ list}$$
$$② = ③$$

$$\beta \text{ list} = \alpha \text{ list} \implies \beta = \alpha \ \Big\}$$
$$\gamma \text{ list} = \alpha \text{ list} \implies \gamma = \alpha \ \Big\} \alpha = \beta = \gamma$$

$$\underset{\tau_1 \ \ \tau_2}{e :: \ell}$$
$$\underbrace{\quad}_{\tau_1 \text{ list}} \quad \tau_1 \text{ list} = \tau_2$$

$$\subseteq [[\ ]]$$

STIK
$$[\ ] @ [\ ] = [\ ]$$

Prednaloga :

$$\underset{\alpha \text{ list}}{[\ ]} :: \underset{\beta \text{ list}}{[\ ]}$$
$$\underbrace{\qquad\qquad}_{(\alpha \text{ list}) \text{ list}} \quad \beta \text{ list} = \alpha \text{ list}$$
$$\Downarrow$$
$$\beta = \alpha$$

glava :: rep

```
let rec map^α =
    fun f^β ->
        fun l^γ ->
            match l^γ with
            | []  -> []^(δ list)
              ε    ε list
            | x :: xs -> f x :: (map f) xs
              ε    ε list    β ε    α  β    ε list
```

$\widetilde{\vartheta}$: $\varepsilon\ list \to \vartheta$

$\widetilde{\eta}$

$\varepsilon \to \eta$

$\underbrace{\beta \to (\varepsilon\ list \to \vartheta)}$

$\underbrace{\qquad}_{\eta\ list}$

$\beta = \varepsilon \to \eta$
$\alpha = \beta \to (\varepsilon\ list \to \vartheta)$
$\vartheta = \eta\ list$

```
let rec map =
    fun f^(ε→η) ->
        fun l^(γ list) ->  (ε list)
            match l^(γ list) with
            | []  -> []^(ϑ list)
              ε    ε list
            | x :: xs -> f x :: (map f) xs
              ε    ε list         α→β  ε→η
                              η      β      ε list
```

$\alpha \to \beta$ $(\varepsilon \to \eta) \to (\varepsilon\ list \to \eta\ list)$

$\underbrace{\qquad}_{\eta\ list}$ (β, ε list)

$\underbrace{\qquad}_{\eta\ list}$

(1) | $\vartheta\ list$ ~~$\vartheta\ list$~~ / $\eta\ list$

(2) |

Rešitev:
$\gamma \longmapsto \varepsilon$
$\vartheta \longmapsto \eta$
$\alpha \longmapsto \varepsilon \to \eta$
$\beta \longmapsto \varepsilon\ list \to \eta\ list$

$\alpha = \varepsilon \to \eta$
$\beta = \varepsilon\ list \to \eta\ list$
$\gamma\ list = \varepsilon\ list \Rightarrow \gamma = \varepsilon$
$\vartheta\ list = \eta\ list = \vartheta = \eta$

(1) $\varepsilon\ list \to \eta\ list$
(2) $(\varepsilon \to \eta) \to \varepsilon\ list \to \eta\ list$

ⓞ DGOVOR : $\underline{\underline{(\varepsilon \to \eta) \to \varepsilon\ list \to \eta\ list}}$

Rekurzija povzroči enačbo :

$(\varepsilon \to \eta) \to (\varepsilon\ list \to \eta\ list) = (\varepsilon \to \eta) \to \varepsilon\ list \to \eta\ list$ ✓