Izpeljava tipov
Vaje

```
fun x -> fun y -> (x, y, y)
```

$x : \alpha$, $y : \beta$, $(x, y, y) : \alpha * \beta * \beta$

$$\alpha \to \beta \to \alpha * \beta * \beta$$

```
fun f -> fun g -> f (g 42)
```

$f : \alpha = \gamma \to \delta$, $g : \beta = \text{int} \to \gamma$, $f\ (g\ 42) : \delta$

$g\ 42 : \gamma$, $g : \beta = \text{int} \to \gamma$, $42 : \text{int}$

$f\ (g\ 42) : \delta$, $f : \alpha = \gamma \to \delta$, $(g\ 42) : \gamma$

$$(\gamma \to \delta) \to (\text{int} \to \gamma) \to \delta$$

Ody: $(\alpha \to \beta) \to (\text{int} \to \alpha) \to \beta$

$h\ e : \gamma$, $h : \tau_2 \to \gamma = \tau_1 \to \tau_2$, $e : \tau_1$

Resitev:

$\beta \mapsto \text{int} \to \gamma$

$\alpha \mapsto \gamma \to \delta$

if e_1 then e_2 else e_3 : $\tau_2 = \tau_3$

```
if 3 < 5 then (fun x -> x) else (fun y -> (y, y))
```

int int bool

$\alpha \to \alpha$

$\beta \to \beta * \beta$

$\alpha \to \alpha$

$\alpha \to \alpha \equiv \beta \to \beta * \beta$ ✓

↳ $\alpha \equiv \beta$ ✓

↳ $\alpha \equiv \beta * \beta$ ✓

↳ $\alpha \equiv \alpha * \alpha$ NI REŠITVE.

Rešitev: $\alpha \mapsto \beta$

$3 < 5$

$3.14 < 5$ $\quad$ $3.1 < 5.7$

$3.1 + 5$ $\quad$ $<$

1. možnost (Ocaml)

$+ : int \to int \to int$ (glavni tip za +)

$+. : float \to float \to float$

2. možnost (SML)

Polimorfizem (ad-hoc)

$+ : int \to int \to int$ } dva tipa, noben ni glavni

$+ : float \to float \to float$ }

Java ima ad-hoc polimorfizem (overloading)

```
class Foo {
    ⋮
    public int f(int x) { ... }
    public bool f(float x, float y) { ... }
}
```

int → int

float × float → bool

$$< \;:\; \alpha \to \alpha \to \text{bool}$$

$(h\,(f\;\;0))\;1$

Naloga 1:

$$\text{fun } h \to \text{fun } f \to h\,\underbrace{(f\;\;0)}_{\alpha}\;\underbrace{1}_{\text{int}}$$

Naloga 2:

$$\text{fun } g\;x \to g\,(g\,(x+3))$$

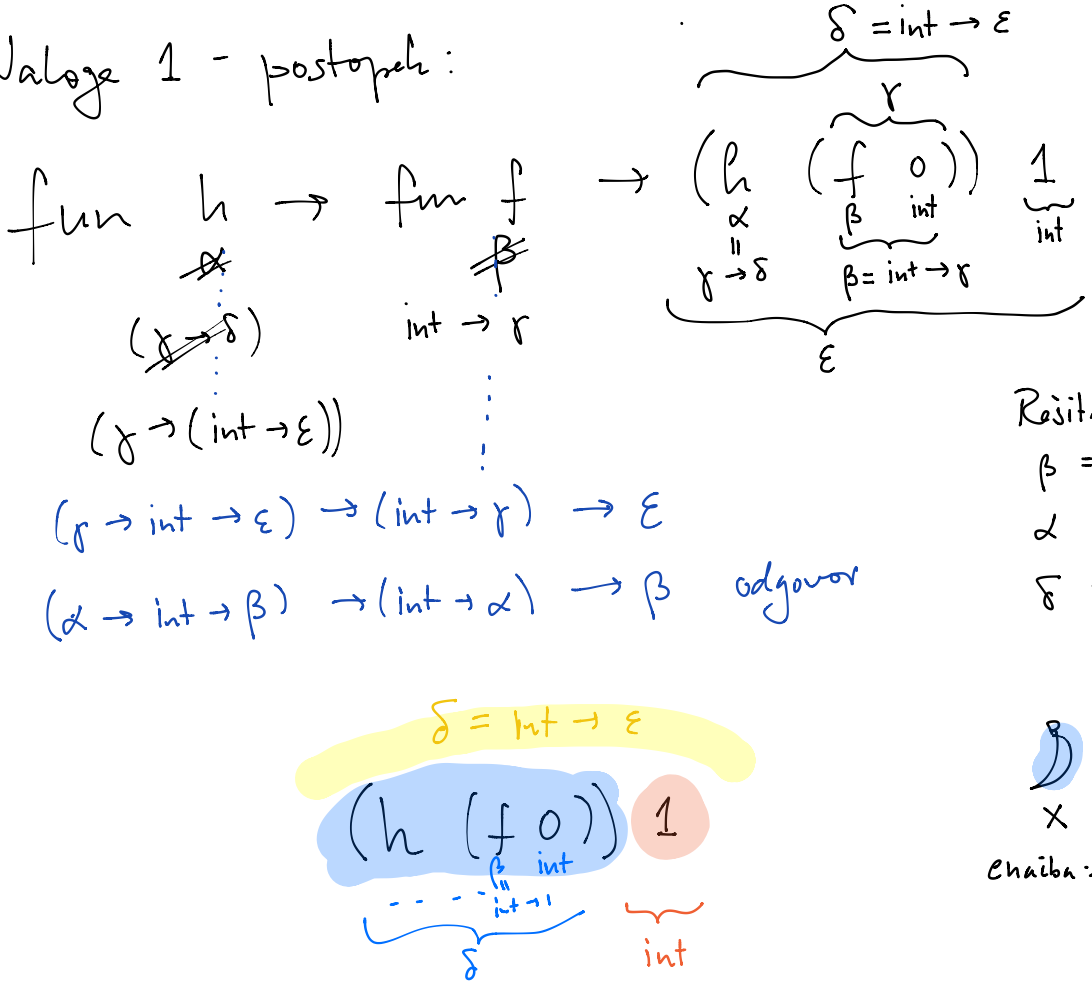
Naloga 1 - rešitev

$((\text{int} \to \beta) \to \text{int} \to \alpha) \to (\text{int} \to \beta) \to \alpha$ (Andrej)

$(\alpha \to (\text{int} \to \beta)) \to ((\text{int} \to \alpha) \to \beta)$ (Gregor)

$(\alpha \to (\text{int} \to \beta)) \to ((\text{int} \to \alpha) \to \beta)$ (Drejc)

Naloga 1 - postopek:



Rešitev:

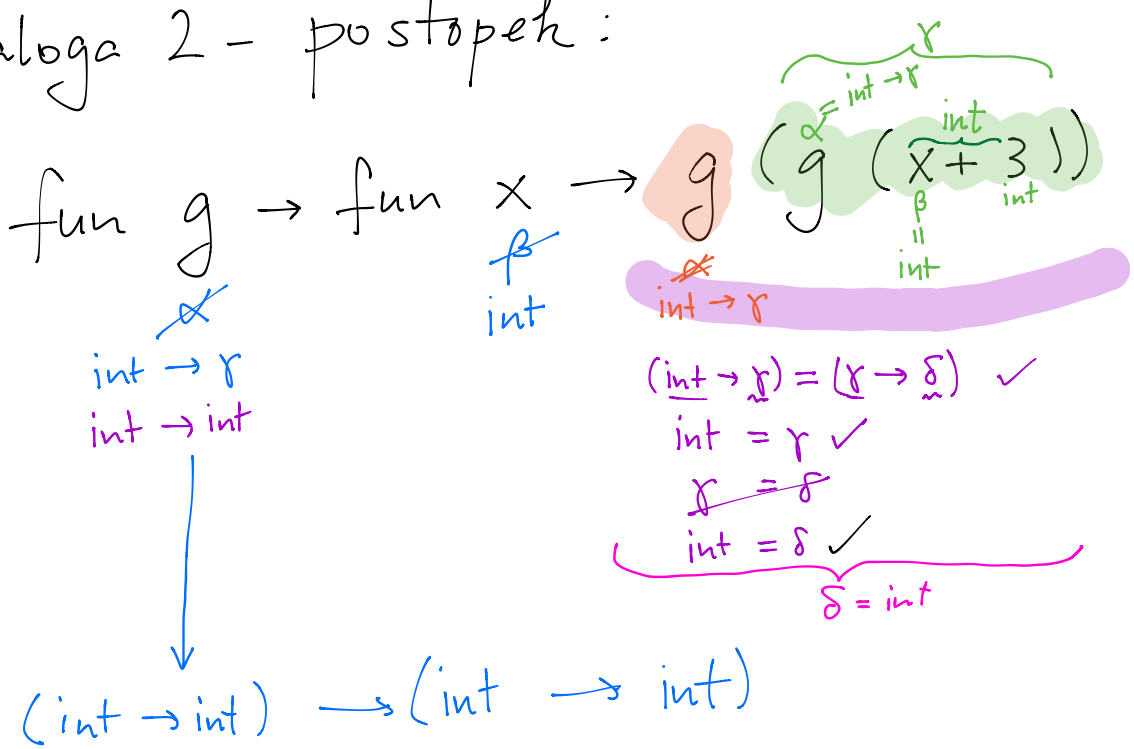
$\beta = int \to \gamma$

$\alpha = \gamma \to \delta$

$\delta = int \to \varepsilon$

enačba: $X = Y \to \alpha$

Naloga 2 - postopek:



Rešitev:

$\beta = int$

$\alpha = int \to \gamma$

$\gamma = int$

$\delta = int$

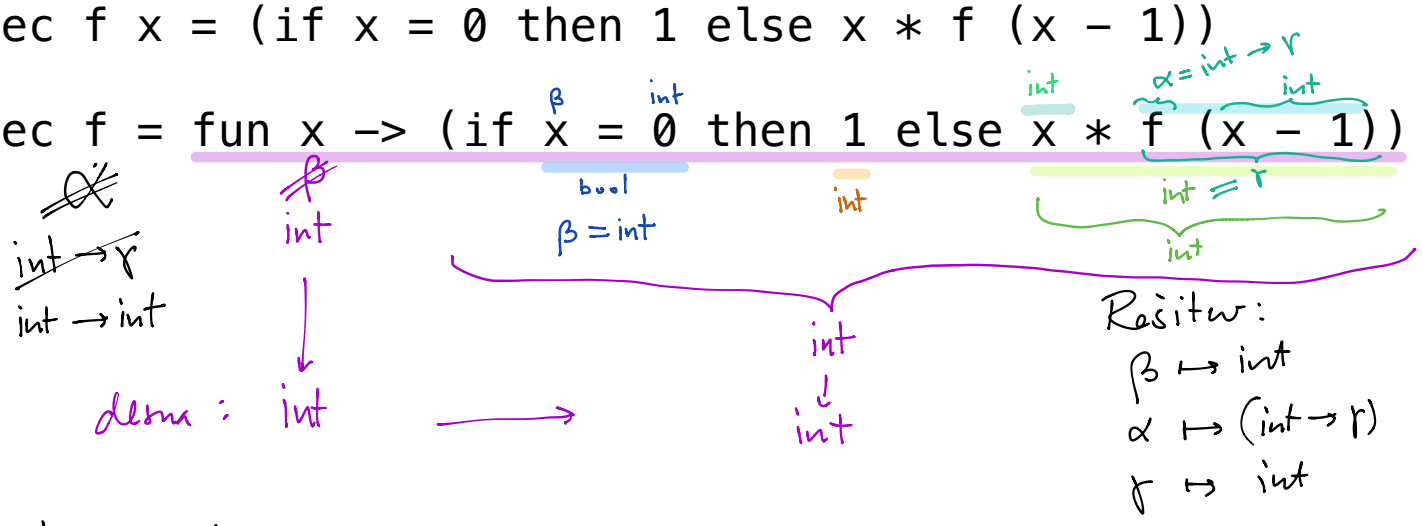
$$(\alpha \to \beta) \to (\alpha * \gamma \to \beta)$$

$$\text{fun } f \to (\text{fun } p \to f\ (\text{fst } p))$$

Naloga:

```
let rec f x = (if x = 0 then 1 else x * f (x - 1))
```

```
let rec f = fun x -> (if x = 0 then 1 else x * f (x - 1))
```



Na koncu: leva = desni

(int → int) = (int → int) ✓

Odgovor: f : int → int

Naloga: Izpelji tip f

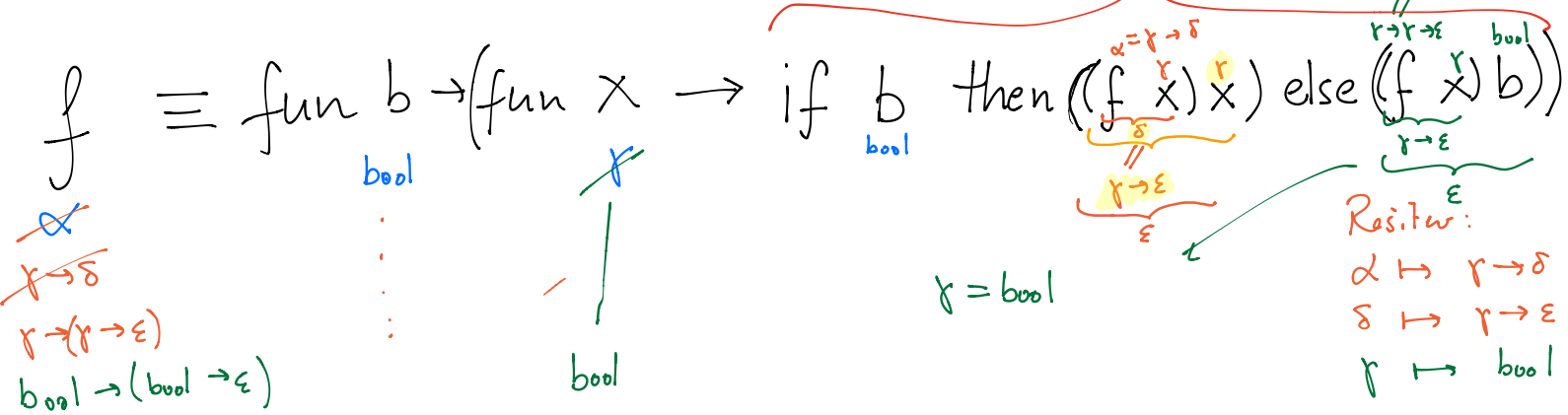
```
let rec f b x = if b then (f x x) else (f x b)
```

$$f \equiv \text{fun } b \to (\text{fun } x \to \text{if } b \text{ then } (f\ x\ x) \text{ else } (f\ x\ b))$$

b: bool, x: bool, b: bool, x: bool, x: bool, b: bool

f: bool → bool → α

bool → bool → α



$bool \to (bool \to \varepsilon) = bool \to (bool \to \varepsilon)$ ✓

$f : bool \to bool \to \varepsilon$