

Ukazni programski jezik

- sintaksa
- operacijska semantika
- (denotacijska semantika)
- prevajalnik

Sintaksa

- cela števila, boolove vrednosti ← izrazi
- spremenljivke, while, if-then-else ← ukazi
- ↑
 celoštevilske

Aritmetični izrazi:

$\langle \text{aritmetični-izraz} \rangle ::= \langle \text{aditivni-izraz} \rangle$

$\langle \text{aditivni-izraz} \rangle ::= \langle \text{multiplikativni-izraz} \rangle \mid \langle \text{aditivni-izraz} \rangle + \langle \text{multiplikativni-izraz} \rangle$

$\langle \text{multiplikativni-izraz} \rangle ::= \langle \text{osnovni-izraz} \rangle \mid \langle \text{multiplikativni-izraz} \rangle * \langle \text{osnovni-izraz} \rangle$

$\langle \text{osnovni-izraz} \rangle ::= \langle \text{spremenljivka} \rangle \mid \langle \text{številka} \rangle \mid (\langle \text{aritmetični-izraz} \rangle)$

$\langle \text{spremenljivka} \rangle ::= [a-zA-Z]^+$

$\langle \text{številka} \rangle ::= -? [0-9]^+$

Boolovi izrazi:

$\langle \text{boolov-izraz} \rangle ::= \text{true} \mid \text{false} \mid$
 $\langle \text{aritmetični-izraz} \rangle = \langle \text{aritmetični-izraz} \rangle \mid$
 $\langle \text{aritmetični-izraz} \rangle < \langle \text{aritmetični-izraz} \rangle \mid$
 $\langle \text{boolov-izraz} \rangle \text{ and } \langle \text{boolov-izraz} \rangle \mid$
 $\langle \text{boolov-izraz} \rangle \text{ or } \langle \text{boolov-izraz} \rangle \mid$
 $\text{not } \langle \text{boolov-izraz} \rangle$

Ukazi:

$\langle \text{ukaz} \rangle ::= \text{skip} \mid$
 $\langle \text{spremenljivka} \rangle := \langle \text{aritmetični-izraz} \rangle \mid$
 $\langle \text{ukaz} \rangle ; \langle \text{ukaz} \rangle \mid$
 $\text{while } \langle \text{boolov-izraz} \rangle \text{ do } \langle \text{ukaz} \rangle \text{ done} \mid$
 $\text{if } \langle \text{boolov-izraz} \rangle \text{ then } \langle \text{ukaz} \rangle \text{ else } \langle \text{ukaz} \rangle \text{ end}$

→ pass (Python)

Pozor: tu ni podpičja!

$C_1 ; C_2$

↑
operacija, ki veže dva ukaza
v enega (podobno $e_1 + e_2$)

Java:

$C_1 ;$
 $C_2 ;$
↑
ločilo, ki označuje
konec ukaza

Prioriteta in asociativnost:

- * `;` (levo)
- * `or` (levo)
- * `and` (levo)
- * `not`
- * `+` (levo)
- * `*` (levo)

$$C_1; C_2; C_3 \equiv (C_1; C_2); C_3$$

$$A \text{ and } B \text{ or } C \equiv (A \text{ and } B) \text{ or } C$$

Primer

```
s := 0;
i := 0;
while i < 101 do
  s := s + i;
  i := i + 1
done
```

Ψ ξ zeta
 ρ ξ ksi \equiv

Operacijska semantika

Okolje: $\eta = [x \mapsto 4, y \mapsto 10, z \mapsto 5]$
 $\eta(x) = 4$
 $\eta[x \mapsto 5]$ okolje η , kjer smo x nastavili na 5
 $\eta[x \mapsto 5] = [x \mapsto 5, y \mapsto 10, z \mapsto 5]$

Okolje je stanje programa med izvajanjem

Aritmetični izrazi:

$$\frac{}{\eta \mid n \hookrightarrow n}$$

$$\frac{\eta(x) = n}{\eta \mid x \hookrightarrow n}$$

$$\eta \mid e_1 \hookrightarrow n_1 \quad \eta \mid e_2 \hookrightarrow n_2$$

$$\frac{\eta \mid e_1 \hookrightarrow n_1 \quad \eta \mid e_2 \hookrightarrow n_2}{\eta \mid e_1 + e_2 \hookrightarrow n_1 + n_2}$$

$$\eta \mid e_1 + e_2 \hookrightarrow n_1 + n_2$$

$$\eta \mid e_1 \hookrightarrow n_1 \quad \eta \mid e_2 \hookrightarrow n_2$$

$$\frac{\eta \mid e_1 \hookrightarrow n_1 \quad \eta \mid e_2 \hookrightarrow n_2}{\eta \mid e_1 - e_2 \hookrightarrow n_1 - n_2}$$

$$\eta \mid e_1 \hookrightarrow n_1 \quad \eta \mid e_2 \hookrightarrow n_2$$

$$\frac{\eta \mid e_1 \hookrightarrow n_1 \quad \eta \mid e_2 \hookrightarrow n_2}{\eta \mid e_1 * e_2 \hookrightarrow n_1 * n_2}$$

Boolovi izrazi:

$$\frac{}{\eta \mid \text{true} \hookrightarrow \text{true}}$$

$$\frac{}{\eta \mid \text{false} \hookrightarrow \text{false}}$$

$b_1 \ \&\& \ b_2$

$$\frac{\eta \mid b \hookrightarrow \text{false}}{\eta \mid \text{not } b \hookrightarrow \text{true}}$$

$$\eta \mid \text{not } b \hookrightarrow \text{true}$$

$$\frac{\eta \mid b \hookrightarrow \text{true}}{\eta \mid \text{not } b \hookrightarrow \text{false}}$$

$$\eta \mid \text{not } b \hookrightarrow \text{false}$$

$$\frac{\eta \mid b_1 \hookrightarrow \text{false}}{\eta \mid b_1 \ \text{and} \ b_2 \hookrightarrow \text{false}}$$

$$\eta \mid b_1 \ \text{and} \ b_2 \hookrightarrow \text{false}$$

$$\eta \mid b_1 \hookrightarrow \text{true} \quad \eta \mid b_2 \hookrightarrow v_2$$

$$\frac{\eta \mid b_1 \hookrightarrow \text{true} \quad \eta \mid b_2 \hookrightarrow v_2}{\eta \mid b_1 \ \text{and} \ b_2 \hookrightarrow v_2}$$

b_1 and b_2 :

1. Izračunamo b_1 in b_2 :

2. Izračunamo b_1 in nato b_2 po potrebi

b_1 and b_2

↑ "stražar" (guard)

Naši izrazi ne spreminijo η .

Java: $(i++ + ++i) * 7$ spremeni i

$$\frac{\eta \mid b_1 \hookrightarrow \text{true}}{\eta \mid b_1 \text{ or } b_2 \hookrightarrow \text{true}}$$

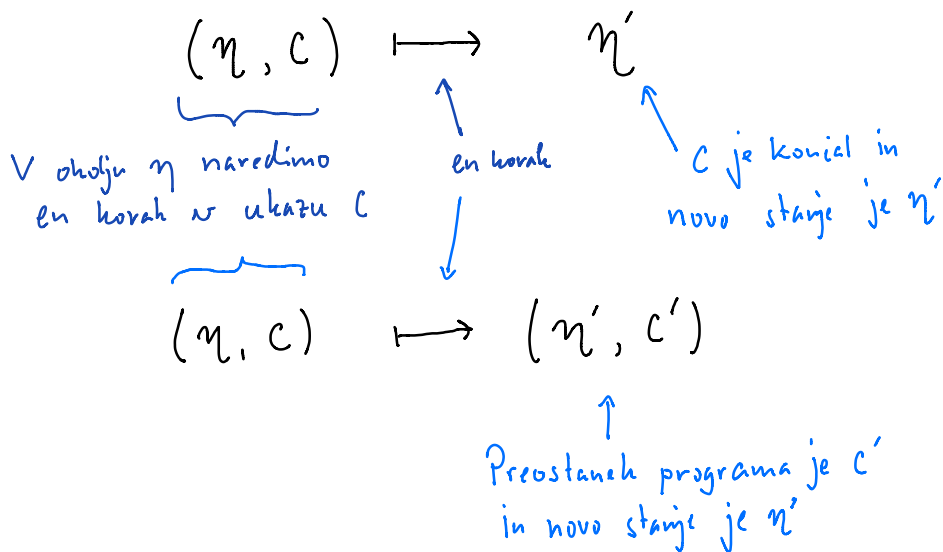
$$\frac{\eta \mid b_1 \hookrightarrow \text{false} \quad \eta \mid b_2 \hookrightarrow v_2}{\eta \mid b_1 \text{ or } b_2 \hookrightarrow v_2}$$

$$\frac{\eta \mid e_1 \hookrightarrow n_1 \quad \eta \mid e_2 \hookrightarrow n_2 \quad n_1 < n_2}{\eta \mid e_1 < e_2 \hookrightarrow \text{true}}$$

$$\frac{\eta \mid e_1 \hookrightarrow n_1 \quad \eta \mid e_2 \hookrightarrow n_2 \quad n_1 \geq n_2}{\eta \mid e_1 < e_2 \hookrightarrow \text{false}}$$

Ukazi: Ukazi spremivjajo okolje

Semantika malih korakov:



$$\overline{(\eta, \text{skip}) \mapsto \eta}$$

$$\frac{\eta \mid e \hookrightarrow n}{(\eta, (x := e)) \mapsto \eta[x \mapsto n]}$$

$$\frac{(\eta, c_1) \mapsto (\eta', c_1')}{(\eta, (c_1 ; c_2)) \mapsto (\eta', (c_1' ; c_2))}$$

$$\frac{(\eta, c_1) \mapsto \eta'}{(\eta, (c_1 ; c_2)) \mapsto (\eta', c_2)}$$

$$\frac{\eta \mid b \hookrightarrow \text{true}}{(\eta, (\text{if } b \text{ then } c_1 \text{ else } c_2 \text{ end})) \mapsto (\eta, c_1)}$$

$$\frac{\eta \mid b \hookrightarrow \text{false}}{(\eta, (\text{if } b \text{ then } c_1 \text{ else } c_2 \text{ end})) \mapsto (\eta, c_2)}$$

$$\frac{\eta \mid b \hookrightarrow \text{false}}{(\eta, (\text{while } b \text{ do } c \text{ done})) \mapsto \eta}$$

$$\frac{\eta \mid b \hookrightarrow \text{true}}{(\eta, (\text{while } b \text{ do } c \text{ done})) \mapsto (\eta, (c ; \text{while } b \text{ do } c \text{ done}))}$$

Primer:

$$\eta = [x \mapsto 5]$$

$$(\eta, (x := x + 3; \text{if } x < 5 \text{ then } x := 10 \text{ else } x := 42 \text{ end})) \mapsto$$

$$([x \mapsto 8], (\text{if } x < 5 \text{ then } x := 10 \text{ else } x := 42 \text{ end})) \mapsto$$

$$([x \mapsto 8], (x := 42)) \mapsto [x \mapsto 42]$$

Denotacijska semantika

Ideja: e sintaktični izraz / ukaz / tip

$\llbracket e \rrbracket$ matematični pomen (denotacija) e

$$\llbracket 42 \rrbracket = 42$$

$\underbrace{\quad}$
niž znakov 4 in 2

\hookrightarrow število kot matematični koncept

$$\llbracket e_1 + e_2 \rrbracket = \llbracket e_1 \rrbracket + \llbracket e_2 \rrbracket$$

\uparrow sintaksa \uparrow operacija sestevanje

Ekvivalenca programov

Kdaj sta dva izraza ali ukaza ekvivalentna?

"ekvivalenca" = "se obnašata enako v vseh situacijah"

Evaluacijski kontekst (situacija) \square

Del programske kode z luknjo \square , v katero lahko vstavimo kodo

Primer:

$E[\] \equiv (\text{if } x < 5 \text{ then } (x := 7; \square) \text{ else skip}$

\uparrow
sem lahko nekaj vstavimo

C_1 in C_2 sta ekvivalentna, kadar za vse evaluacijske kontekste

$E[\]$ in okolja η velja:

$\left. \begin{array}{l} (\eta, E[C_1]) \\ (\eta, E[C_2]) \end{array} \right\} \begin{array}{l} \text{bodisi oba se izvajata v vedogled,} \\ \text{bodisi se oba končata v istem konicnem} \\ \text{stanju} \end{array}$

Primer:

$x := x + 1 ;$
 $x := x + 2$

je ekvivalentno

$x := x + 3$

Primer:

$x := x + 1 ;$
 $x := x + 2$

in

$y := y + 3$

nista ekvivalentna



$$\eta = [x \mapsto 0, y \mapsto 0]$$

$$(\eta, (x := x + 1; x := x + 2)) \mapsto \dots \mapsto [x \mapsto 3, y \mapsto 0]$$

$$(\eta, (y := y + 3)) \mapsto [x \mapsto 0, y \mapsto 3]$$